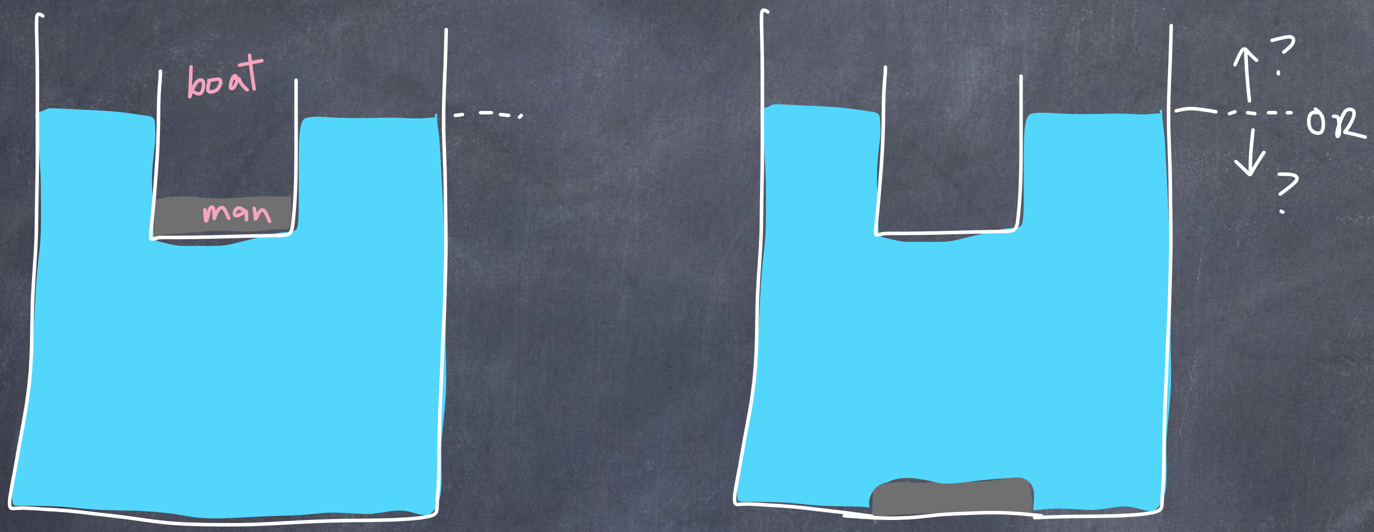
PHY 117 HS2023

Week 5, Lecture 2
Oct. 18th, 2023
Prof. Ben Kilminster

Man overboard !



Level goes down. Why?

On the left, the volume displaced is more than on the right. The displaced fluid raises the fluid level on the left. When the boat is empty, it is less dense and displaces less fluid.

So level goes down.

Fluids In Motion!

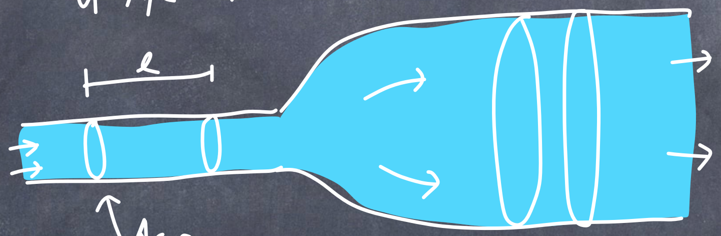
pipe with fluid inside

volume $V_2 = A_2 v_2 \Delta t$

$V_1 = A_1 \ell = A_1 v_1 \Delta t$

velocity of fluid v_1 incoming

ℓ



v_2: velocity outgoing

volume V coming in time Δt

Area A_1

area, A_2

Since fluids are incompressible, $V_1 = V_2$

$$A_1 v_1 \cancel{\Delta t} = A_2 v_2 \cancel{\Delta t}$$

$$A_1 v_1 = A_2 v_2 = \text{constant}$$

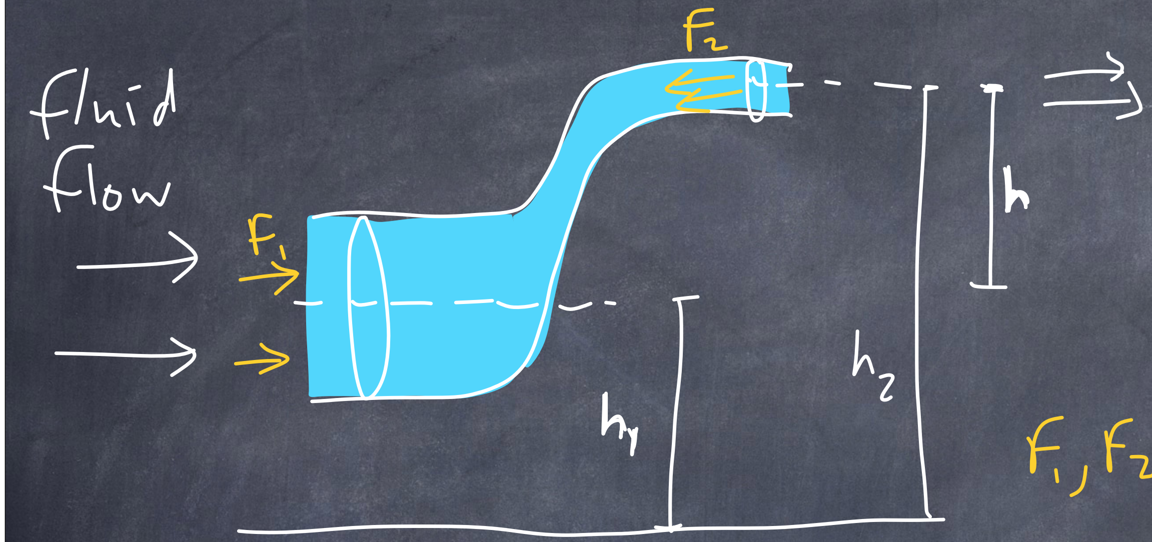
(tube changes area)

$$I_V \equiv A v : \left[m^2 \frac{m}{s}\right] \left[\frac{m^3}{s}\right]$$ Volume flow rate

$$\boxed{I_V = vA = \text{constant}}$$ continuity equation

If A gets bigger, then v gets smaller

What if it changes height?



fluid flow

$$h = h_2 - h_1$$

F_1, F_2 are due to fluid pressure

Fluid gain potential energy, must lose kinetic energy.

In some time Δt, some $\underbrace{\text{amount of fluid}}_{\Delta m:\ \text{mass}}$ gets lifted by a height, h.

Change in potential energy $= \Delta U = \Delta m g h = \rho \Delta V g h$

ΔV: volume of fluid being lifted

ΔK = change in kinetic energy $= \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2$

$$= \frac{1}{2}\rho\Delta V\left(v_2^2 - v_1^2\right)$$

The work-energy theorem states that

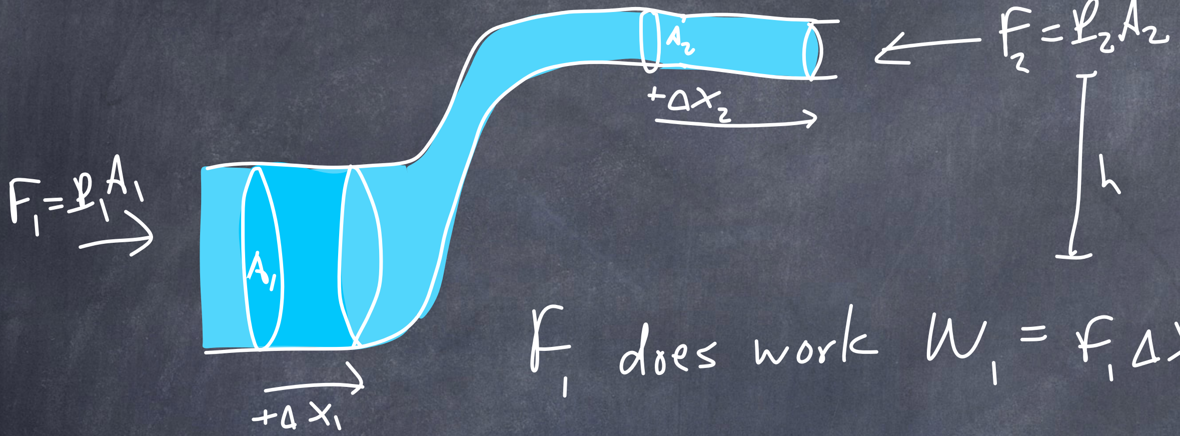
$$\boxed{W_{TOTAL} = \Delta U + \Delta K}$$

work done by fluid

$$W_{TOTAL} = \rho\Delta V g h + \frac{1}{2}\rho\Delta V\left(v_2^2 - v_1^2\right) \quad (1)$$

we know that work comes from a force times a distance. The force, f_1 comes from pressure, P_1 and f_2 comes from pressure at top, P_2.

What if it changes height ?



Fluid is pushed with F_1 by fluid pressure to its left and pushed back with F_2 by the fluid to the right

F_1 does work $W_1 = F_1 \Delta x_1 = P_1 \underbrace{A_1 \Delta x_1}_{\text{volume of cylinder}} = P_1 \Delta V$

F_2 does work $W_2 = (-F_2) \Delta x_2 = -P_2 \Delta V$

same volumes

The total work done is $W_{TOTAL} = W_1 + W_2$

$= P_1 \Delta V - P_2 \Delta V$

$$\boxed{W_{TOTAL} = (P_1 - P_2)\, \Delta V} \quad ②$$

we combine ① + ②

$$(P_1 - P_2) \cancel{\Delta V} = \rho \cancel{\Delta V} g h + \frac{1}{2} \rho \cancel{\Delta V} (v_2^2 - v_1^2)$$

$$P_1 - P_2 = \rho g(h_2 - h_1) + \frac{1}{2}\rho(v_2^2 - v_1^2)$$

we move our terms:

$$\underbrace{P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2}_{\text{position 1}} = \underbrace{P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2}_{\text{position 2}}$$

In other words,

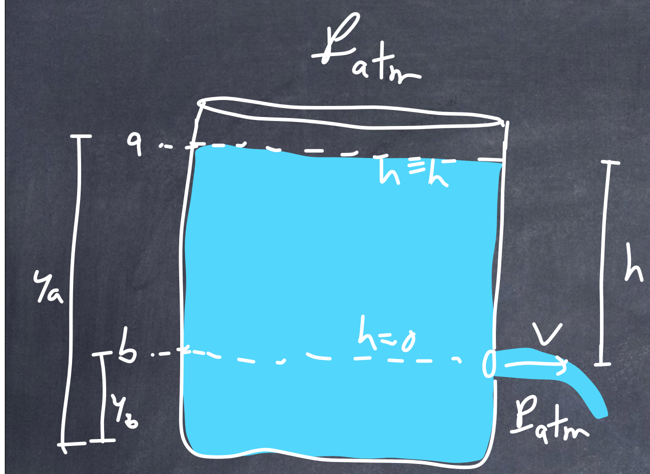
$$\boxed{P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}}$$

Bernoulli's equation.

- every term has units of pressure.
- neglects friction

→ This combination of quantities stays constant while height, area, velocity, U, K, P changes.

This is a statement of energy conservation.



$P_a = P_b = P_{atm}$

Approximation: $v_a = 0$.
Valid if the opening at the top is much larger than the hole at a.

Tank with a hole in it, open at top.

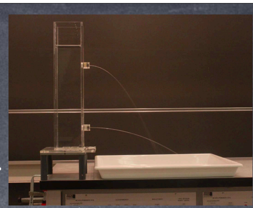


Figure out the velocity of water
we apply Bernoulli's equation at heights a+b.

<u>Level a</u>   <u>Level b</u>

$$P_a + \rho g h + \tfrac{1}{2}\rho v^2 = P_b + \rho g h + \tfrac{1}{2}\rho v^2$$

$$\cancel{P_{atm}} + \rho g h + \tfrac{1}{2}\rho \cancel{v_a}^{2} = \cancel{P_{atm}} + \rho g \cancel{h} + \tfrac{1}{2}\rho v_b^2$$

$$\cancel{\rho} g h = \tfrac{1}{2}\cancel{\rho} v_b^2$$

$v_b = v$ of water exiting the hole.

$$\boxed{v = \sqrt{2gh}}$$

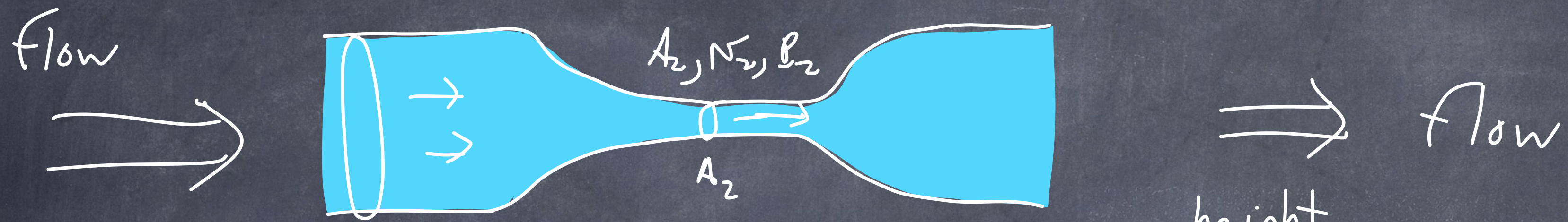
Torricelli's Law

Imagine an object falling a distance x.

$v^2 = v_0^2 + 2ax \Rightarrow$ If $a = g$, $v_0 = 0$, $x = h$

$\Rightarrow v^2 = 2gh \Rightarrow v = \sqrt{2gh}$

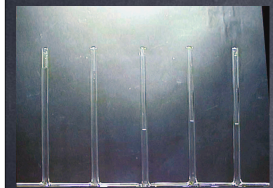
Fluid moving in a pipe with changing area



A_1

A_1, v_1, P_1

height doesn't change



we know that $A_1 v_1 = A_2 v_2$ so $v_2 = v_1 \left(\frac{A_1}{A_2}\right)$

Bernoulli's equation becomes: $P + \frac{1}{2}\rho v^2 = \text{constant}$

(constant height)

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

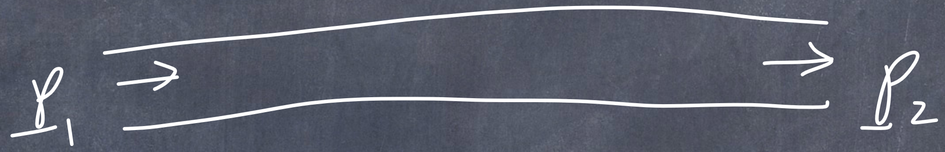
since $v_2 > v_1$, then it must be that

$$P_2 < P_1$$

Venturi effect — when the speed of a fluid increases, then the pressure gets smaller.

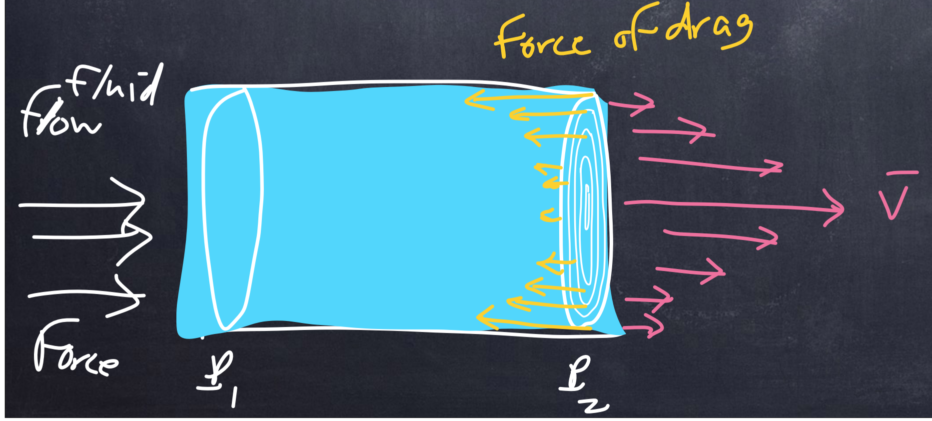
Current of Fluid, I_V moves with "viscous" flow.

Bernoulli's equation states that the pressure is the same anywhere in a pipe at constant height and area.

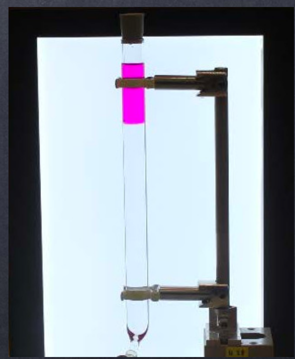


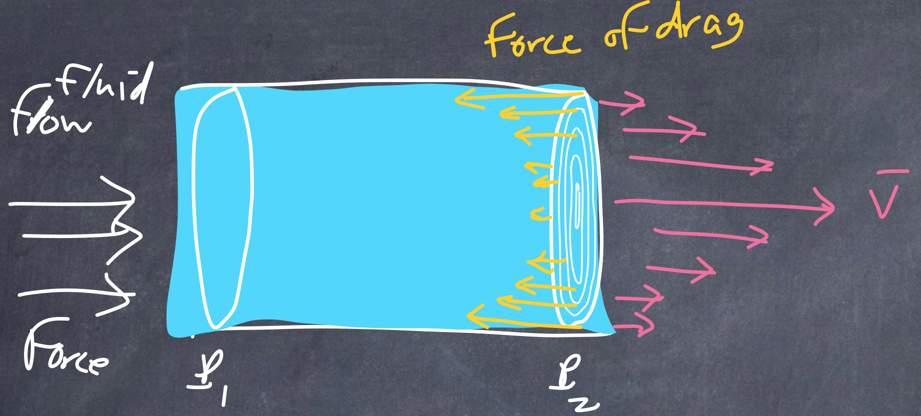
$P_1 = P_2$

In practice, we see a pressure drop. The pressure drop comes from a drag force from the surface of the pipe on the fluid, but also from each layer of fluid on the next layer.



velocity is faster in center of tube.





velocity faster in center

Pressure drop from P_1 to P_2

$$P_1 - P_2 = I_v R = (v A) R$$

velocity of the fluid

area of the pipe

constant of resistance

Pressure difference $P_1 - P_2 = \underbrace{I_v}_{I_v = vA} \underbrace{R}_{\text{constant of resistance}}$

The resistance for steady flow in a cylindrical pipe is

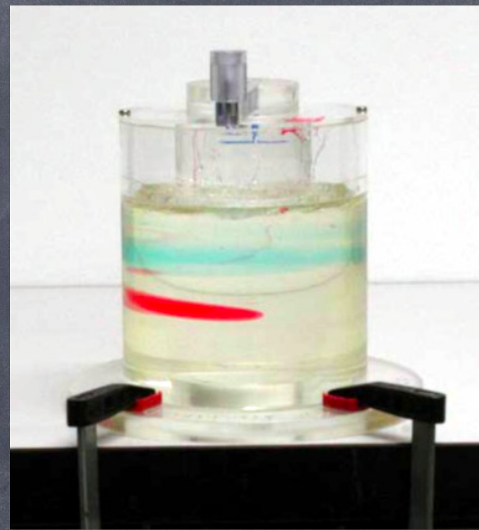
$$R = \frac{8 \eta L}{\pi r^4}$$

L: length of the pipe
r: radius of the pipe
η: coefficient of viscosity.

η: has units of $\left[\frac{N \cdot s}{m^2}\right] = [Pa \cdot s]$

$$1 \, Pa \cdot s = 10 \text{ Poise}$$

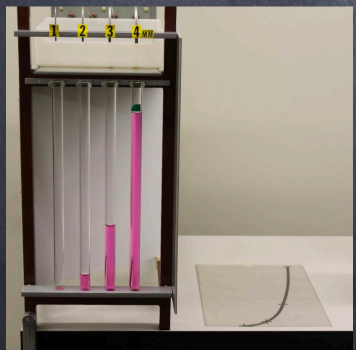

a



For our pipe,

$$\Delta P = I_V R = I_V \left(\frac{8 \eta L}{\pi r^4} \right)$$

so $$\underbrace{I_V}_{vA} = \Delta P \left(\frac{\pi r^4}{8 \eta L} \right)$$



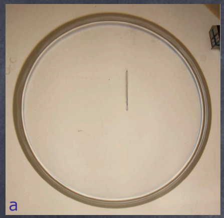
$I_V \sim r^4$

$I_V \sim \frac{1}{L}$

For constant A,

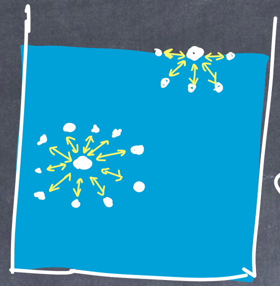
$v \sim \frac{1}{L}$

surface tension



molecules are attracted to each other

$\underbrace{\text{to each other}}_{\text{a cohesive force.}}$



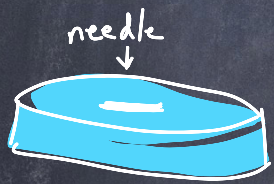
← Here we see an unbalanced force at top

← Here, equal force from all directions.

surface tension is the unbalanced force from the cohesive force of molecules below, which makes the surface like a stretchable membrane.

side view

surface is like a stretched membrane.



There is a restoring force F_{ST}, proportional to the total length of object

$$F_{ST} = \gamma L$$

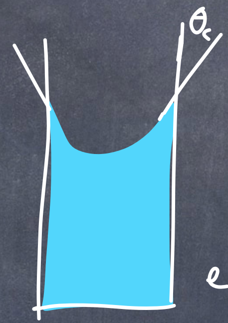
where γ is a constant that depends on the fluid + its temperature and gas in contact.

$\gamma = 0.073 \frac{N}{m}$ for water at room temperature.

One consequence of this is <u>capillary action</u>.

This comes from the adhesive + cohesive forces. The adhesive force is between the fluid and the walls of the container.

θ_c : contact angle



If adhesive forces > cohesive forces.

This shape is known as the meniscus.

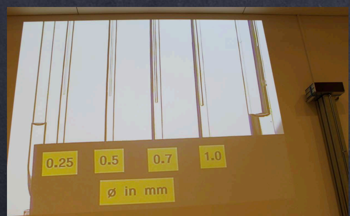
e.g. water + glass

$\theta_c = 0^\circ$

If adhesive forces < cohesive forces,

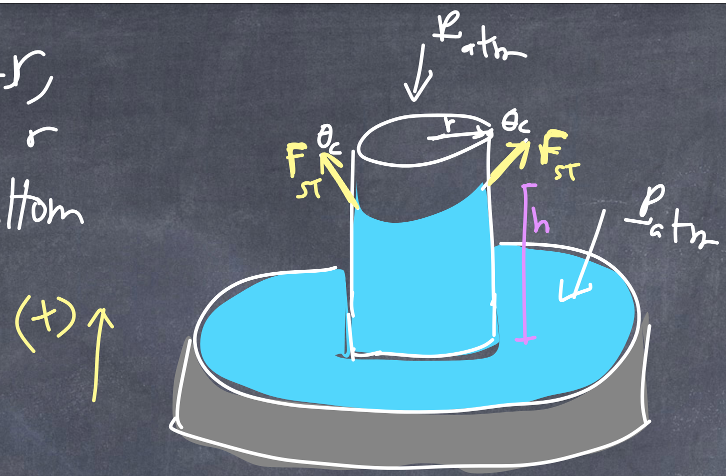
e.g. mercury + glass, ($\theta_c = 140^\circ$)

θ_c : measure, depends on the fluid and the container



Consider cylinder, radius r

open on top + bottom



Adhesive force pulls fluid upward

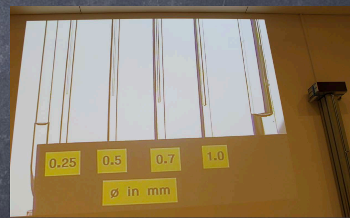
F_{ST} : adhesive force pulling upward.

$$h = \frac{2\gamma\cos\theta_c}{\rho r g}$$

: the height that the adhesive force raises a fluid in a container.

when $\theta_c < 90^\circ$, $\cos\theta_c > 0 \Rightarrow h$ is (+)

when $\theta_c > 90^\circ$, $\cos\theta_c < 0 \Rightarrow h$ is (−)



Derivation of height formula in the backup slides.

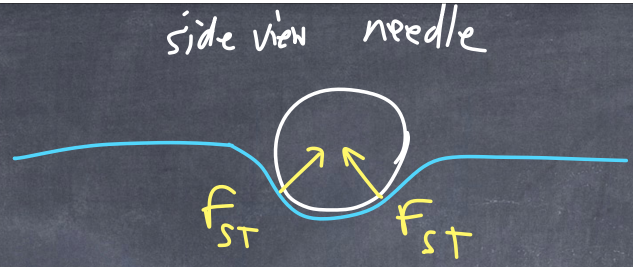
end

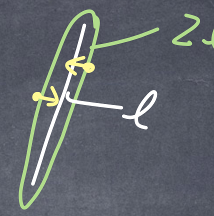
After this, there are a few derivations for your information.

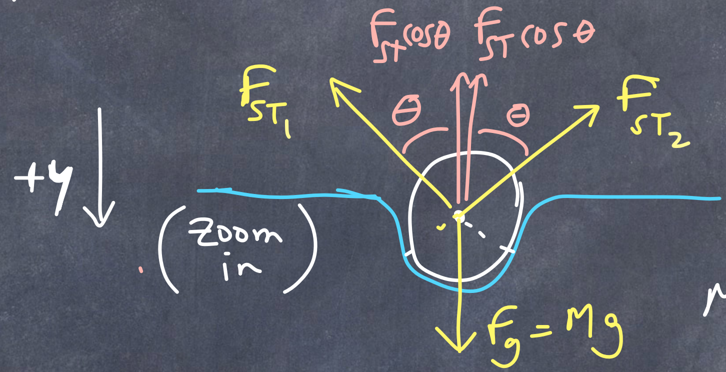
There is a surface force on both sides of the needle, so

$$F_{ST} = \gamma L = \gamma 2\ell$$

ℓ, length of the needle







$\longleftrightarrow$ in horizontal direction, $F_{ST} \rightarrow$ these forces cancel out.

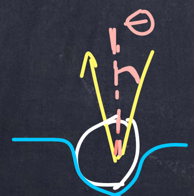
M: total mass of needle

In the y-direction, the total surface tension is

$$F_{ST_y} = \overset{(\gamma\ell\cos\theta)}{F_{ST_1}\cos\theta} + \overset{(\gamma\ell\cos\theta)}{F_{ST_2}\cos\theta} = 2F_{ST}\cos\theta = 2\gamma\ell\cos\theta$$

The needle floats as long as $F_{ST_y} > F_g$

M small



M big

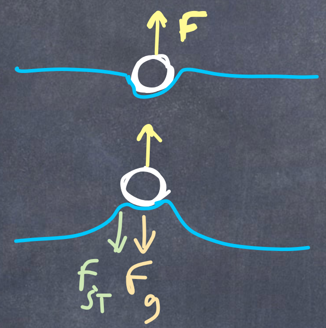
As M gets larger, θ decreases.

When $\theta = 0°$, $\cos\theta = 1 \Rightarrow F_{ST} = 2\gamma\ell$

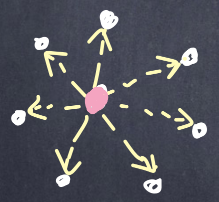
The maximum mass allowed is when

$Mg = 2\gamma\ell\cos 0°$ $\quad m_{max} = 2\gamma\ell/g$

The force to lift the needle off the surface is $F = mg + \gamma 2L$



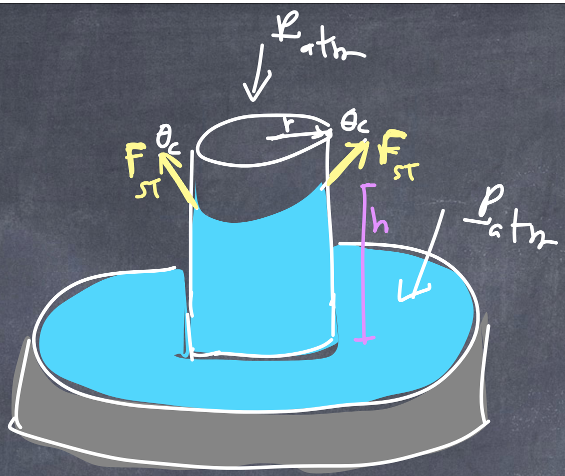
In this case, the surface tension resists us pulling the needle up, because we are stretching the fluid membrane upward.



cohesive force on one molecule ● is coming from the surrounding molecules.

Consider cylinder, radius r

open on top + bottom



Adhesive force pulls fluid upward

F_{ST} : adhesive force pulling upward

vertical direction $\left\{ \sum F = F_{ST}\cos\theta_c - mg = 0 \right.$

$$\gamma L \cos\theta_c = mg$$

$$\gamma\, 2\pi r \cos\theta_c = \rho V g$$

$$\gamma\, 2\cancel{\pi}\cancel{r}\cos\theta_c = \rho(\cancel{\pi} r^{\cancel{2}} h) g$$

what is L? It's the length of contact between fluid + container

$$L = 2\pi r$$

$$h = \frac{2\gamma\cos\theta_c}{\rho r g}$$: The height that the adhesive force raises a fluid in a container

when $\theta_c < 90°$, $\cos\theta_c > 0 \Rightarrow h$ is (+)

when $\theta_c > 90°$, $\cos\theta_c < 0 \Rightarrow h$ is (−)