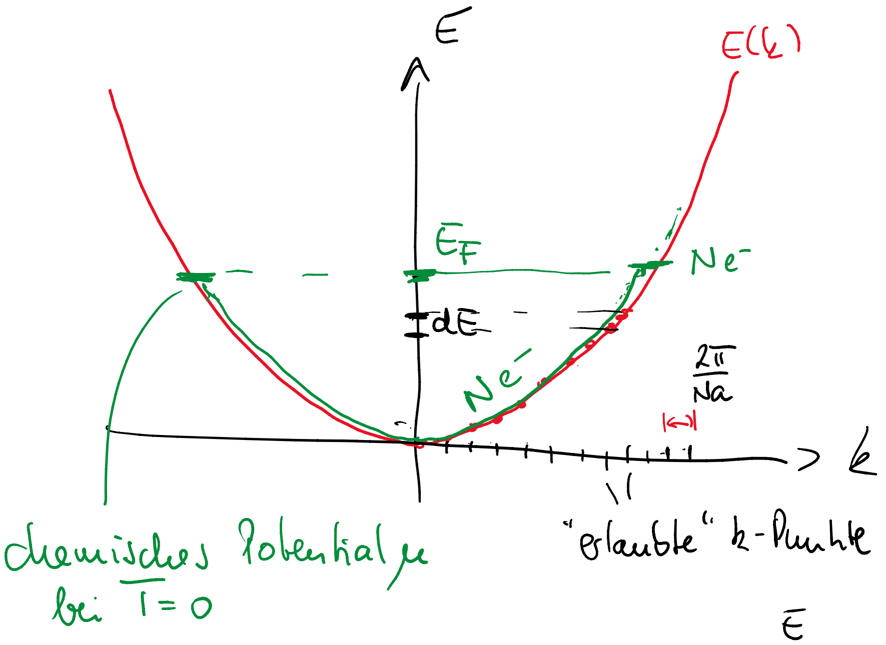
Übungen 27-35

Mittwoch, 15. November 2023 17:36

Afg 28: Zustandsdichte und Fermi-Energie

Metall in 1D, 2D



$$E = \frac{\hbar k^2}{2m_e}$$

N Punkte in 1. BZ

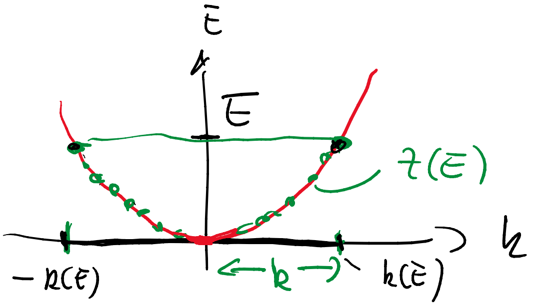
Elektronen: 1 pro Atom
dh N Elektronen

Zustandsdichte D(E)? $\int_0^E D(\tilde E)\, d\tilde E = Z(E)$

$\underline{D(E)}\, dE$ = Zahl von Zuständen zwischen E und E+dE

$= \underline{\frac{dZ}{dE}}\, dE$ Z Zahl der Zustände mit Energie < E

Z(E)



$$Z(E) = n_k \cdot 2 \cdot \underbrace{\boxed{\iiint d^3k}}$$

Anzahldichte k-Punkten

Spins ↑↓

besetztes Volumen $\tilde E < E$

$\tilde E = \frac{\hbar^2 k^2}{2m}$

Volumen in d Dimension

Punktdichte im k-Raum: $\boxed{\frac{V_d}{(2\pi)^d} = n_k}$ d= Dimension

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1D:

$$\Rightarrow Z(E) = \frac{V_d}{(2\pi)^d} = \frac{L}{\cancel{2\pi}}\,\cancel{2}\int_{-k(E)}^{+k(E)} dk$$ L= Länge

$$= \frac{L}{\pi}\cdot 2k(E) = \frac{L}{\pi}\cdot 2\sqrt{\frac{2m_eE}{\hbar^2}}$$

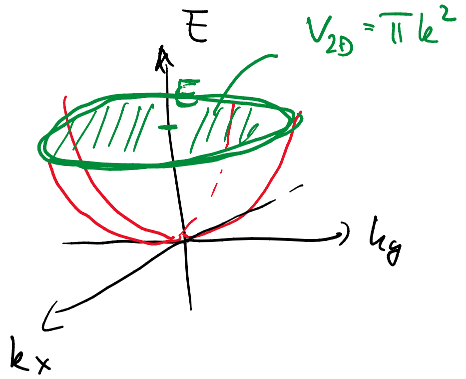
$$= \frac{2L}{\pi\hbar}\sqrt{2m_e}\cdot\sqrt{E}$$

$$\boxed{D(E) = \frac{dZ}{dE} = \frac{2L}{\pi\hbar}\sqrt{2m_e}\,\frac{1}{2\sqrt{E}} \sim \frac{1}{\sqrt{E}}}$$

E_F? $\quad Z(E_F) \overset{!}{=} \underline{N}$

$$L = \frac{2L}{\pi\hbar}\sqrt{2m_e}\,\sqrt{\underline{E_F}}$$

$$E_F = \left(\underbrace{\frac{N}{L}}_{n_e}\frac{\pi}{2}\right)^2\frac{\hbar^2}{2m_e} = n_e^2\,\frac{\pi^2\hbar^2}{8m_e}$$ $E_F \sim n_e^2$



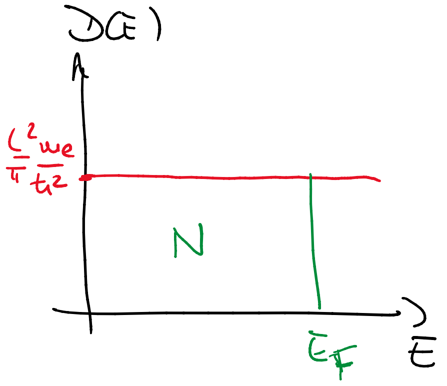
2D:

"Volumen" des Kreisballs

$$Z(E) = \frac{L^2}{(2\pi)^2}\cdot\underset{\text{Spins}}{2}\cdot\underline{\pi k^2(E)}$$

$$= \frac{L^2}{2\pi}\,\frac{2m_eE}{\hbar^2}$$

$$D(E) = \frac{dZ}{dE} = \frac{L^2}{\pi}\,\frac{m_e}{\hbar^2} = \text{konstant!}$$



$$Z(E_F) = \frac{L^2}{2\pi}\,\frac{2m_eE_F}{\hbar^2} = N$$

$$\Rightarrow E_F = \frac{\pi\hbar^2}{m_e}\cdot n$$ $E_F \sim n_e$

$$\Rightarrow \quad E_F = \frac{\pi \hbar^2}{m_e} \cdot n \qquad\qquad E_F \sim n_e$$

Afg 29: Fermi-Gase in 3D

$$\boxed{E_F = \frac{\hbar^2}{\underset{\uparrow}{2m}} \underbrace{\left(3\pi^2 n\right)}_{k_F^3}{}^{2/3}} \qquad\qquad k_F = \sqrt[3]{3\pi^2 n}$$

a) Na Metall: 1 3s-Elektron pro Atom

$\rho = 0.97 \frac{g}{cm^3}$ $\qquad M_{mol} = 23 \frac{g}{mol}$

$$n = \frac{\rho}{M_{mol}} \cdot N_A = \frac{\#\text{Atome}}{cm^3} = \frac{\# e^-}{cm^3} = 2.54 \times 10^{28}\, m^{-3}$$

$\Rightarrow m = m_e = 9.1 \times 10^{-31}\, kg$

$\Rightarrow E_F = 5.1 \times 10^{-19}\, J = \underline{\underline{3.16\, eV}}$

$T_F = \frac{E_F}{k_B} = 36'634\, K.$ $\qquad T = 300\, K \sim k_B T \simeq 25\, meV$

$v_F = \sqrt{\frac{2E}{m}} = 10^6\, m/s$

b.) $^3He = 2p + 1n + 2e^-$ (Spin $\frac{1}{2}$)

$n_{He} = 1.65 \times 10^{28}\, m^{-3}$

$m_{He} = 2 \cdot m_p + 1 m_n = 5 \times 10^{-27}\, kg$

$\Rightarrow E_F = 4.3 \times 10^{-4}\, eV$

$T_F = 5\, K$

$v_F = 166\, m/s$

c) Neutronenstern $\quad m_n = 1.5\, M_\odot = 3 \times 10^{30}\, kg$

c.) Neutronenstern $m_{Stern} = 1.5 M_\odot = 3 \times 10^{30}\,kg$

$R = 15'000\,m$

$$\Rightarrow \quad \rho = 2 \times 10^{17} \frac{kg}{m^3}$$

$$n_n = 1.28 \times 10^{44}\,m^{-3}$$

$$E_F = 5 \times 10^7\,eV \sim 50\,MeV \qquad (m_n c^2 \sim 939\,MeV)$$

$$T_F = 6 \times 10^{11}\,K$$

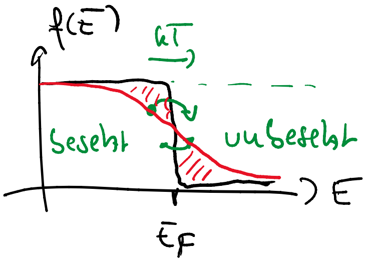
$$v_F = 9.85 \times 10^7\,m/s \cong \frac{c}{3}$$

Afg 33: Spezifische Wärme Cu

$$C_v = \underbrace{\gamma T}_{El.} + \underbrace{\beta T^3}_{Phononen} \qquad \text{bei } T \to 0$$

$$\gamma = 0.69 \frac{J}{mol\,K^2} \quad \text{aus der Kurve}$$

$$= \boxed{\frac{\pi^2}{3} k_B^2 \; D(E_F) = \gamma}$$



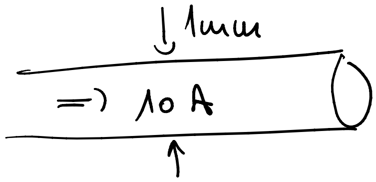
3D Metall $1e^-$ pro Atom

$$D(E_F) = \frac{3N\,m_e}{\hbar^2} \left(3\pi^2 n\right)^{-2/3}$$

$$\Rightarrow \quad m_e = 1.25 \times 10^{-30}\,kg = \underline{\underline{1.37\,m_e}}$$

Afg 34: Widerstand und Geschwindigkeiten in Cu

a.) Driftgeschwindigkeit



n_e $$\dot{j} = \frac{I}{A} = n_e \cdot v_D$$

$$I = \frac{Q}{t} = \underbrace{\frac{Q}{A \cdot l}}_{n} A \cdot \underbrace{\frac{l}{t}}_{v}$$

Drift $v \hat{=} 1\,mm/s \ll v_F = 10^6 m/s$

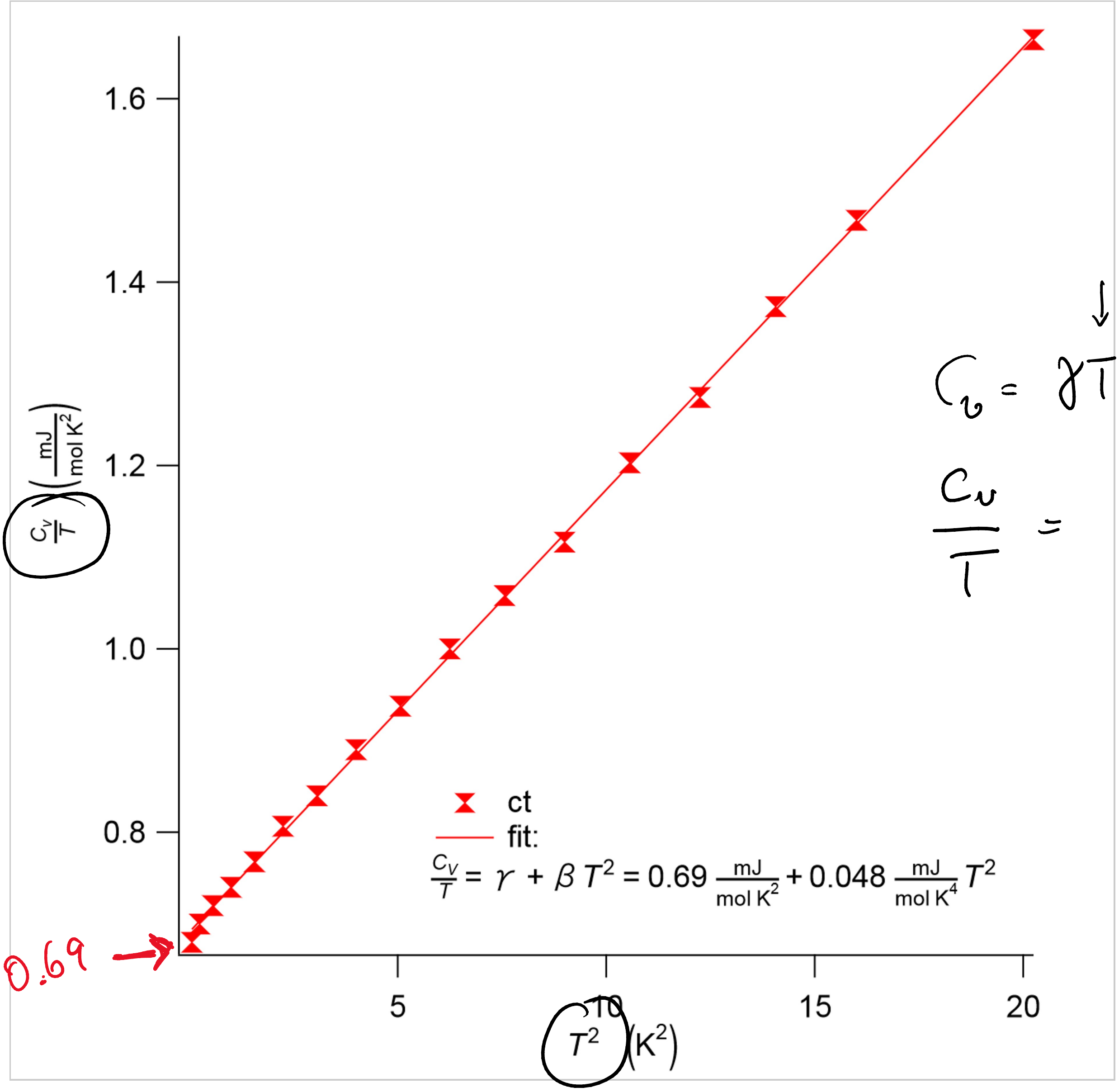
$\uparrow$ Stössen

b) Mittl. freie Weglänge = Weg zwischen Stössen

$$\sigma = \frac{n_e e^2}{v_F m_e} l_{imp} \Rightarrow l_{imp} = 38.6\,nm$$

UE 33 Spezifische Wärme

Donnerstag, 16. November 2023 11:20



0.69 →

$$C_v = \overset{\downarrow}{\gamma T} + \overset{\downarrow}{\beta T^2}$$

$$\frac{C_v}{T} = \underset{4}{\gamma} + \beta T^2$$