PHY117 HS2023

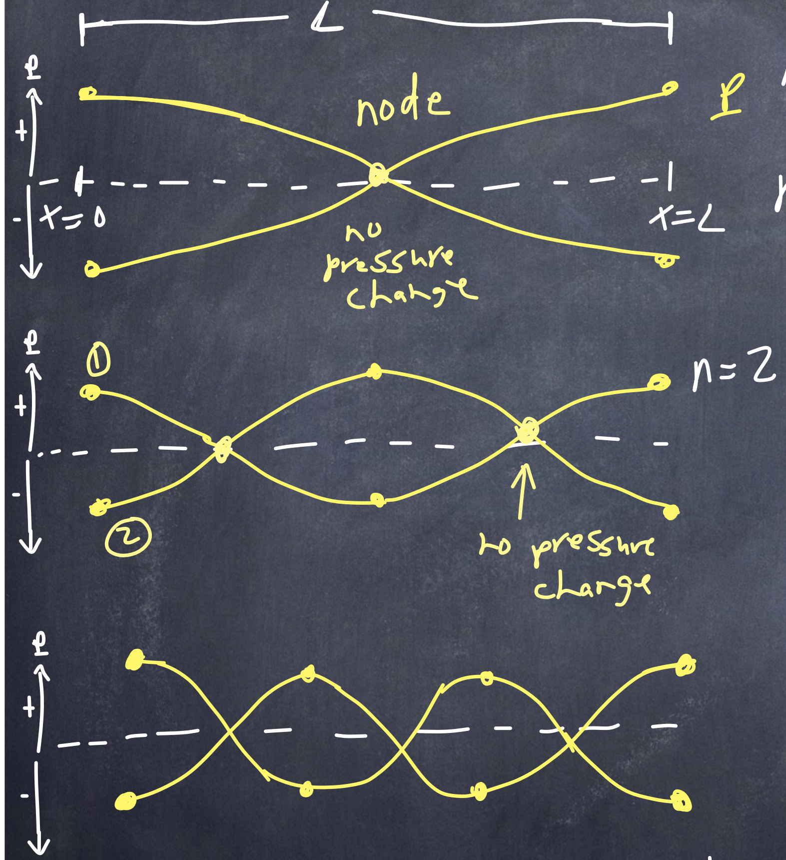
Week 11, Lecture 2

Dec. 6th, 2023

Prof. Ben Kilminster

Standing sound waves in a closed tube of length L

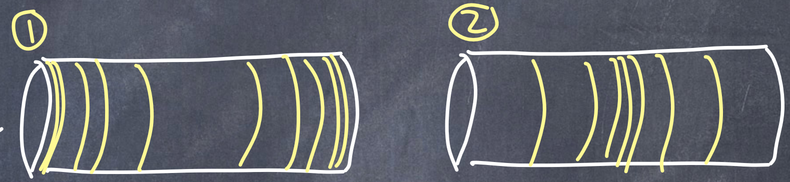
The pressure is an anti-node at both ends.



$n=1 \qquad L = \frac{1}{2}\lambda_1 \qquad f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$

$n=2 \qquad L = \lambda_2 \qquad f_2 = \frac{v}{\lambda_2} = \frac{v}{L}$

$n=2$



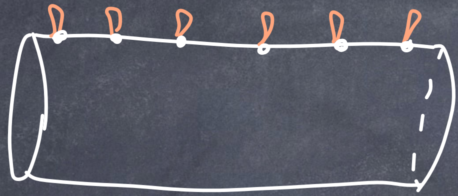
$n=3 \qquad L = \frac{3}{2}\lambda_3 \qquad f_3 = \frac{v}{\lambda_3} = \frac{3}{2}\frac{v}{L}$

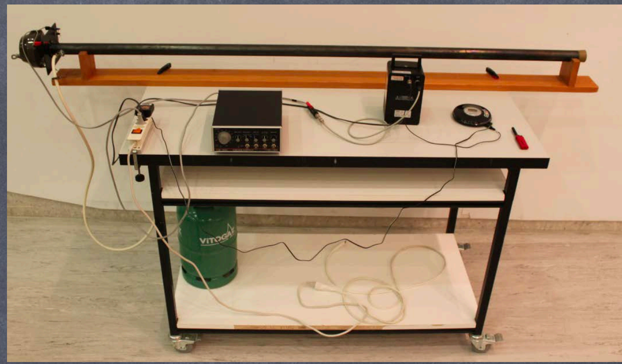
In general, $L = \frac{n}{2}\lambda_n \qquad f_n = \frac{v}{\lambda_n} = \frac{nv}{2L}$

so $\lambda_n = \frac{2L}{n}$ for $n=1,2,3,\ldots$

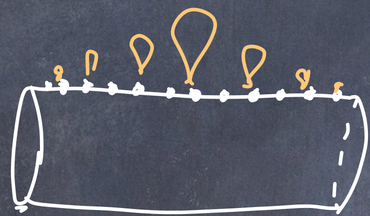
Ruben's flame tube

1





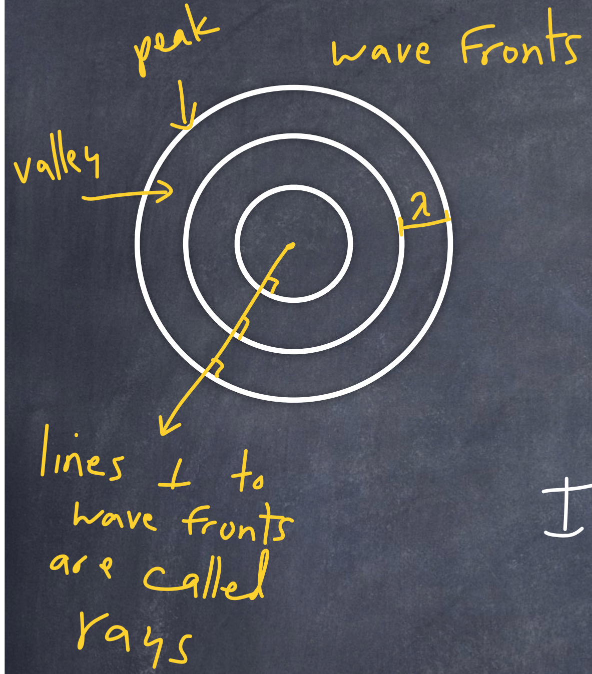
n=1

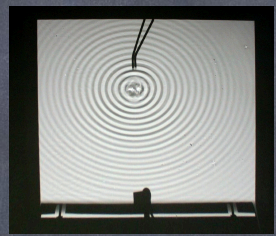


n=2

n=1 & n=2

Waves moving in 2 or 3 dimensions

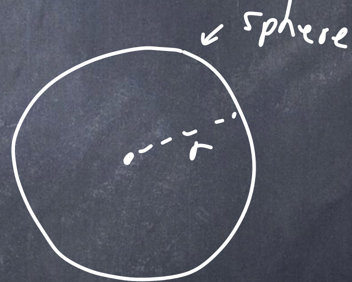




Consider 2-D circular wave from a point source (like throwing stone in water)

Energy moves in all directions uniformly

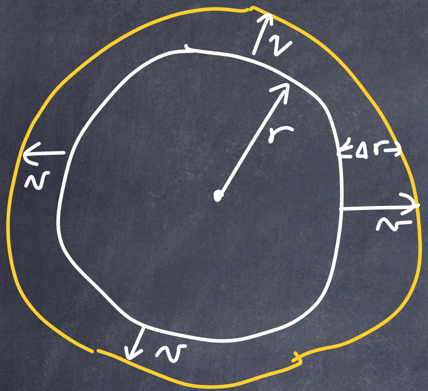
In 3-D, like sound waves, the wave moves spherically.



surface area of a 3-D sphere is $4\pi r^2$

$$\text{Intensity} = I = \frac{\langle \text{power} \rangle}{\text{area}}$$ ← average power

Relationship between I and $\underbrace{\text{energy per unit volume}}_{\eta = \frac{\epsilon}{V}}$



$\Delta r = v \Delta t$

The additional energy in this spherical shell of thickness Δr

$$\Delta \epsilon = \eta \underbrace{\Delta V}_{\text{volume of shell}} = \eta \underbrace{A \Delta r}_{\Delta V} = \eta A v \Delta t$$

$$\text{Power} = \frac{\Delta \epsilon}{\Delta t} = \eta A v$$

$$\boxed{\text{Intensity} = \frac{\text{Power}}{\text{area}} = \eta v}$$

velocity of wave

energy density

$$\text{units} = \left[\frac{W}{m^2}\right]$$

we remember for a s.h.o., $E = \frac{1}{2} m \omega^2 A^2$

$$\Delta E = \frac{1}{2} \underbrace{\Delta m}_{\rho \Delta V} \omega^2 \underbrace{A^2}_{S_0^2}$$

S_0^2: maximal displacement

$$\Delta E = \frac{1}{2} \rho \Delta V \omega^2 S_0^2$$

$$\eta = \frac{\Delta E}{\Delta V} = \frac{1}{2} \rho \omega^2 S_0^2$$

$$I = \eta v = \frac{1}{2} \rho \omega^2 S_0^2 v$$

for sound waves: $P_0 = \rho \omega v S_0$

so $S_0^2 = \frac{P_0^2}{\rho^2 \omega^2 v^2}$

$$\boxed{I = \frac{1}{2} \frac{P_0^2}{\rho v}}$$

Intensity of sound waves

"How loud is something"

Human ear can hear intensities from:

$$I = 10^{-12}\ \frac{W}{m^2} \longrightarrow 1\ W/m^2$$

$\underbrace{10^{-12}\ \frac{W}{m^2}}_{\text{breathing}}$ $\underbrace{1\ W/m^2}_{\text{painful}}$

rock concer, 2m from speakers.

So we use a log scale to measure "loudness" = sound intensity

$$\boxed{\text{Sound intensity level} = \beta = 10 \log_{10} \frac{I}{I_0}}$$

measured in decibels [dB]

$$I_0 = 10^{-12}\ \frac{W}{m^2}$$

rules for $\log_{10}$:

$\log_{10} 1 = 0$

$\log_{10} 10 = 1$

$\log_{10} 100 = 2$

$\log_{10} 1000 = 3$

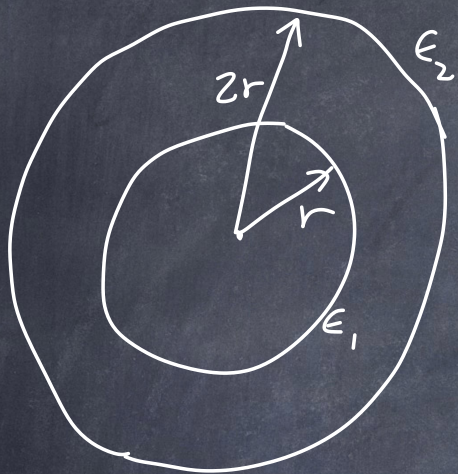
(# of zeroes)

breathing: $\beta = 10 \log \dfrac{10^{-12}\ \frac{W}{m^2}}{10^{-12}\ \frac{W}{m^2}} = 10 \log 1 = 0\ dB$

rock concert $\beta = 10 \log \dfrac{1 \frac{W}{m^2}}{10^{-12} \frac{W}{m^2}} = 10 \log_{10} 10^{12} = 120\ dB$

me talking $\beta = 10 \log 10^6 = 60\ dB$

For 3-D waves: $I = \dfrac{\text{power}}{\text{area}}$ ← for sphere: $4\pi r^2$



So Intensity decreases like $\dfrac{1}{r^2}$

If we know the intensity at r, what is intensity at $2r$?

$$r_1 = r$$
$$r_2 = 2r$$

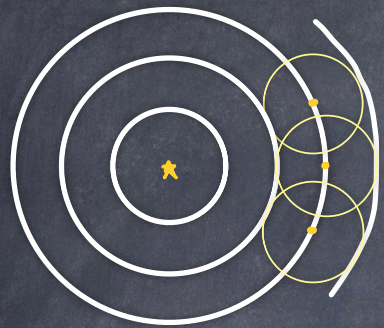
total power at r_1 = total power at r_2

$$I_1(4\pi r_1^2) = I_2(4\pi r_2^2)$$

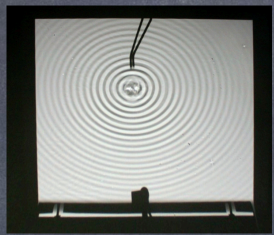
So if $r_2 = 2r_1$

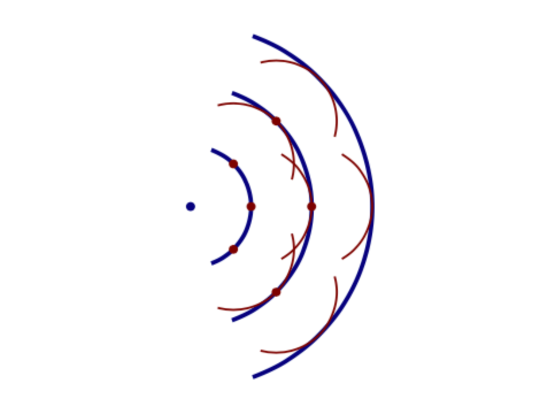
then $I_2 = \frac{1}{4} I_1$

Huygen's principle - a wave acts as if each point along the wave front is a spherical wave source.



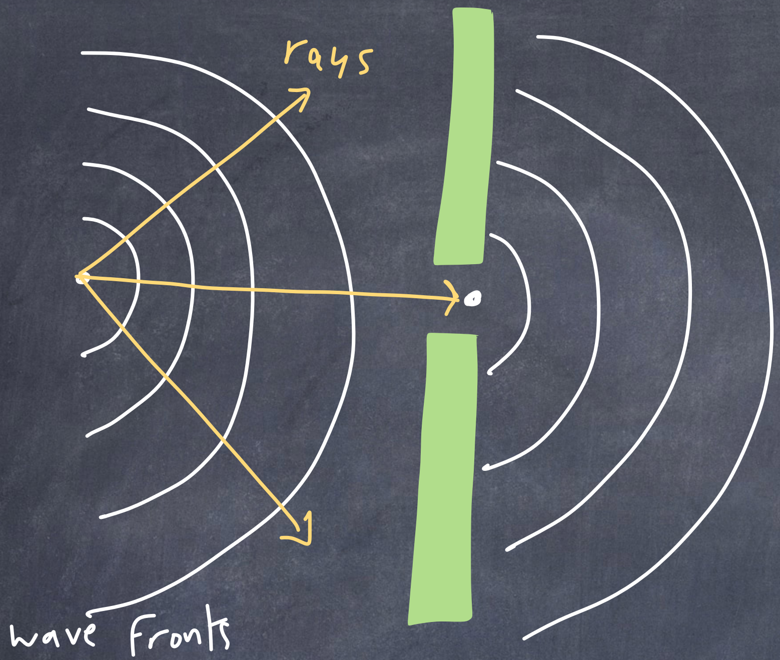
← represents the wave front produced by all point sources along wave front.



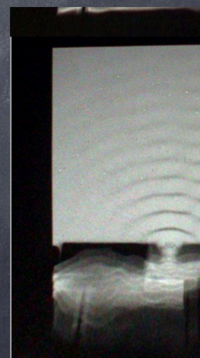


(b) Each point on a wavefront is a new point source.

A plane wave is comprised of harmonic spherical waves

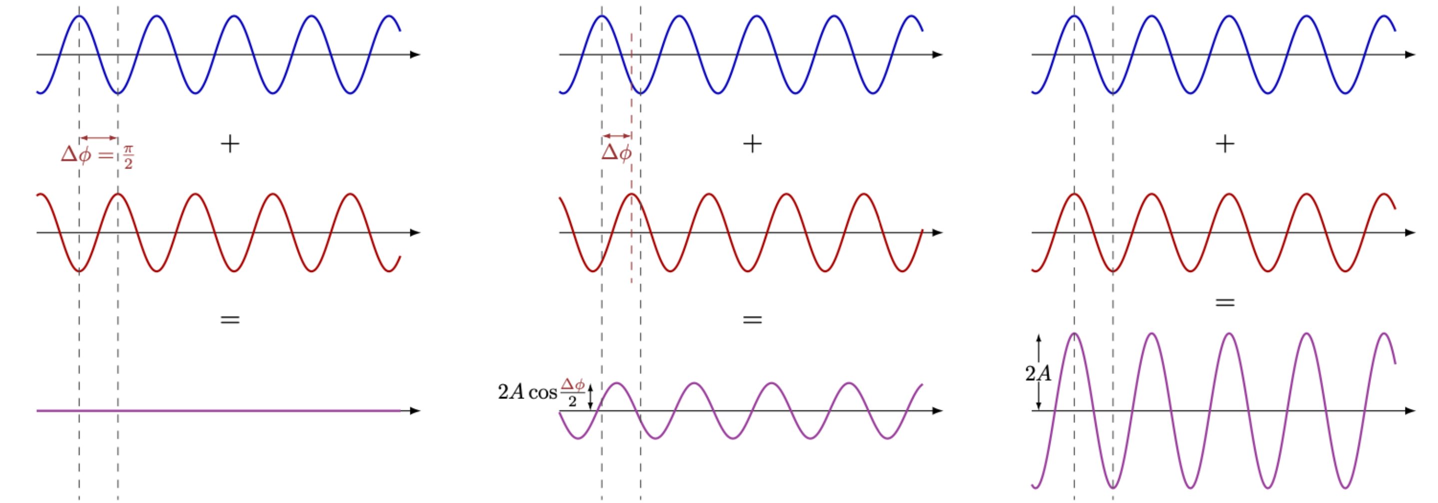


wave propagates as if from one point



Interference:

wave superposition can lead to interference if waves of same λ & amplitude are out of phase by some angle $\Delta\phi$



(a) If $\Delta\phi = 90°$, there is destructive interference.

(b) If $0° < \Delta\phi < 90°$, there is partial interference with amplitude $2A\cos\frac{\Delta\phi}{2}$.

(c) If $\Delta\phi = 0°$, there is constructive interference with amplitude $2A$.

Figure 14.11: Interference between two waves with the same wavelength λ and amplitude A, but phase difference $\Delta\phi$.

Adding 2 waves out of phase

$$y_1 = y_0 \sin(kx - \omega t)$$

$$y_2 = y_0 \sin(kx - \omega t + \delta)$$

δ = phase shift (angle)

$$y_1 + y_2 = 2y_0 \cos\left(\tfrac{1}{2}\delta\right)\sin\left(kx - \omega t + \delta/2\right)$$

Example for pressure waves

$$\mathcal{P}_1 = \mathcal{P}_0 \sin(kx - \omega t)$$

$$\mathcal{P}_2 = \mathcal{P}_0 \sin(kx - \omega t + \delta)$$

$$\mathcal{P}_1 + \mathcal{P}_2 = 2\mathcal{P}_0 \cos\left(\tfrac{1}{2}\delta\right)\sin\left(kx - \omega t + \delta/2\right)$$

One way to get a phase difference is to have different path lengths.

sources are coherent: same k, ω, λ, amplitude, in phase

receiving a signal here

Source1

source 2

phase difference

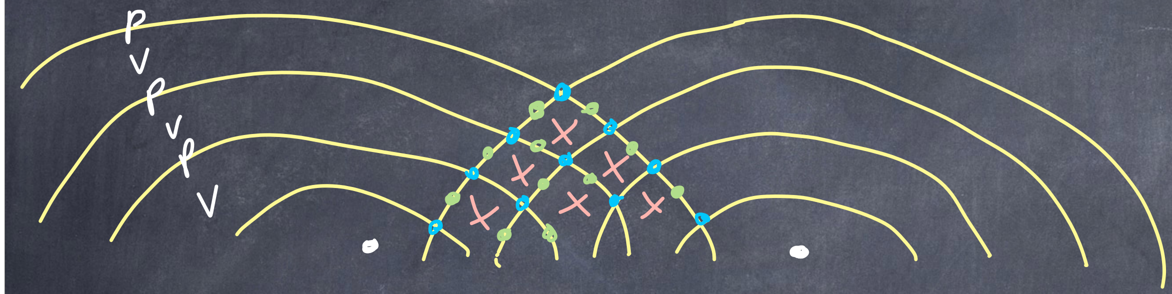
$$\delta = (kx_1 - \omega t) - (kx_2 - \omega t)$$
$$= kx_2 - kx_1 = k\Delta x$$

Since $k = \frac{2\pi}{\lambda}$, so $\boxed{\delta = \frac{2\pi\Delta x}{\lambda}}$ phase difference from 2 coherent sources

If $\Delta x = n\lambda$, $n = 0, 1, 2, \ldots$

then $\delta = 2\pi n$, and we get constructive interference.

2 sources coherently emitting waves



waves add together constructively when peaks overlap

waves adding together at valleys constructively

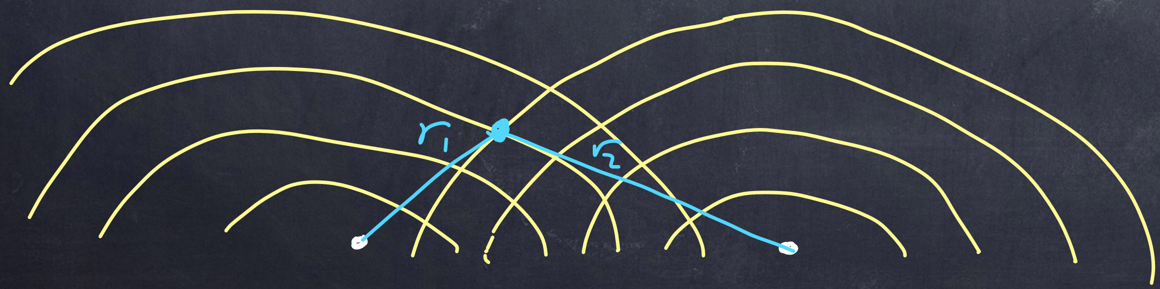
waves add destructively (peak of one wave + valley of the other wave.)

Interference:

constructive when $y_1 + y_2 = 2y_0$

destructive when $y_1 + y_2 = 0$

condition for constructive interference

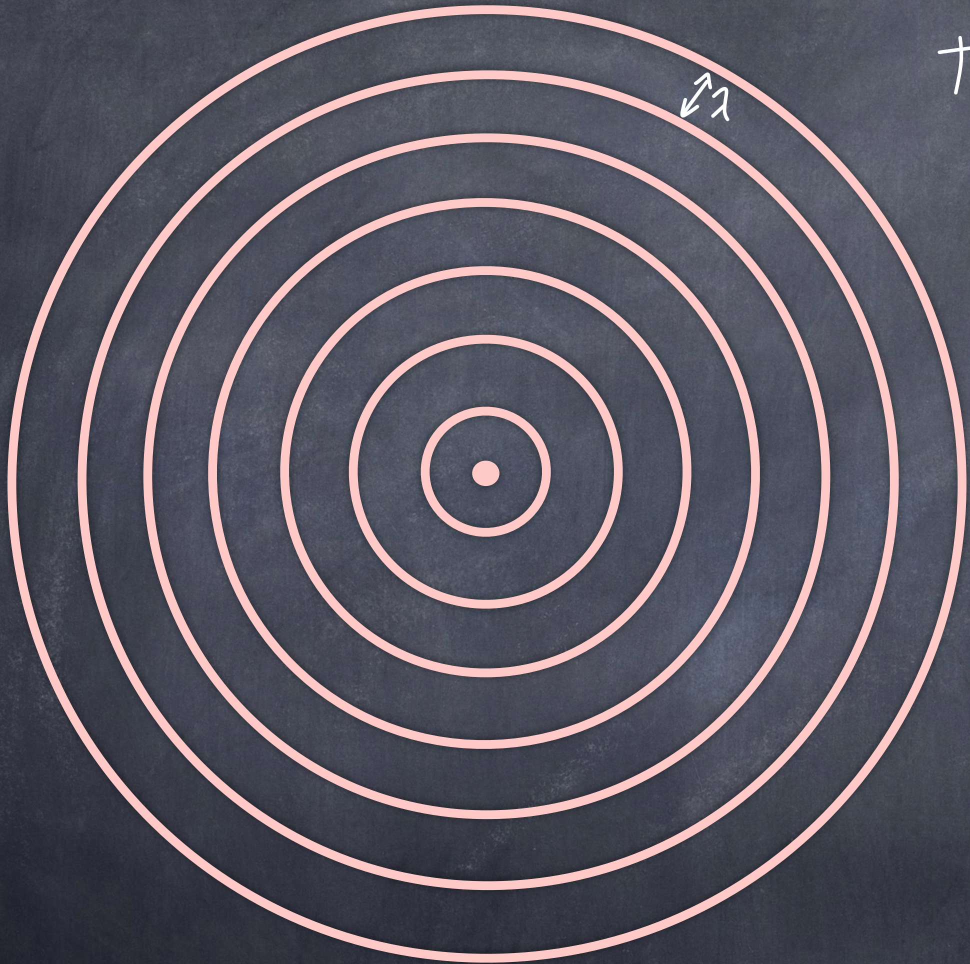


$$\Delta r = r_2 - r_1 = \begin{array}{c} -1\lambda \\ -2\lambda, \\ 0\lambda, \\ 1\lambda \\ 2\lambda \end{array}$$

then we get constructive interference

Here is a source emitting a spherical wave.
The distance between peaks is λ.

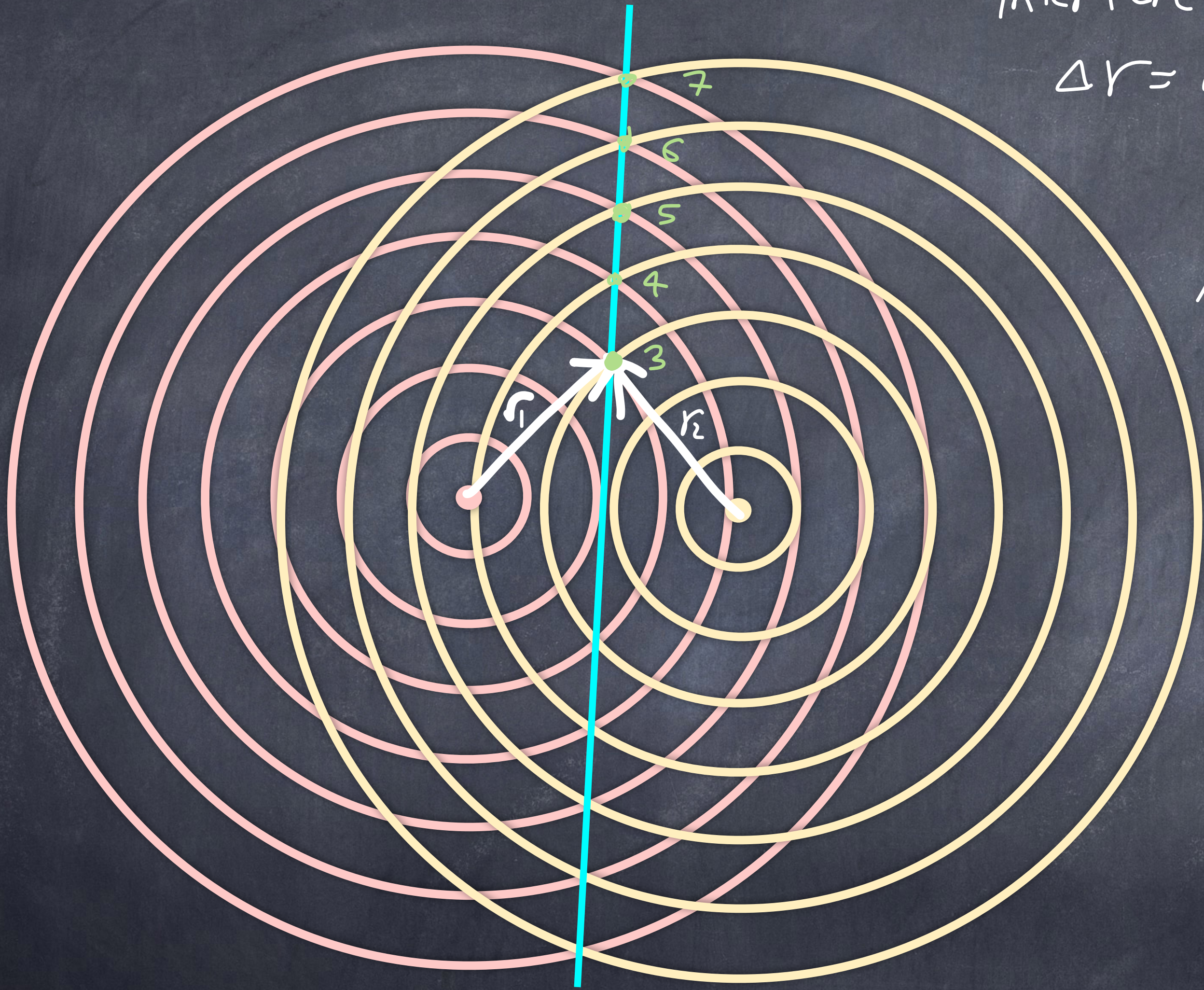
this is a s.h.o.



Here are 2 coherent sources: same ω, k, in phase at source.

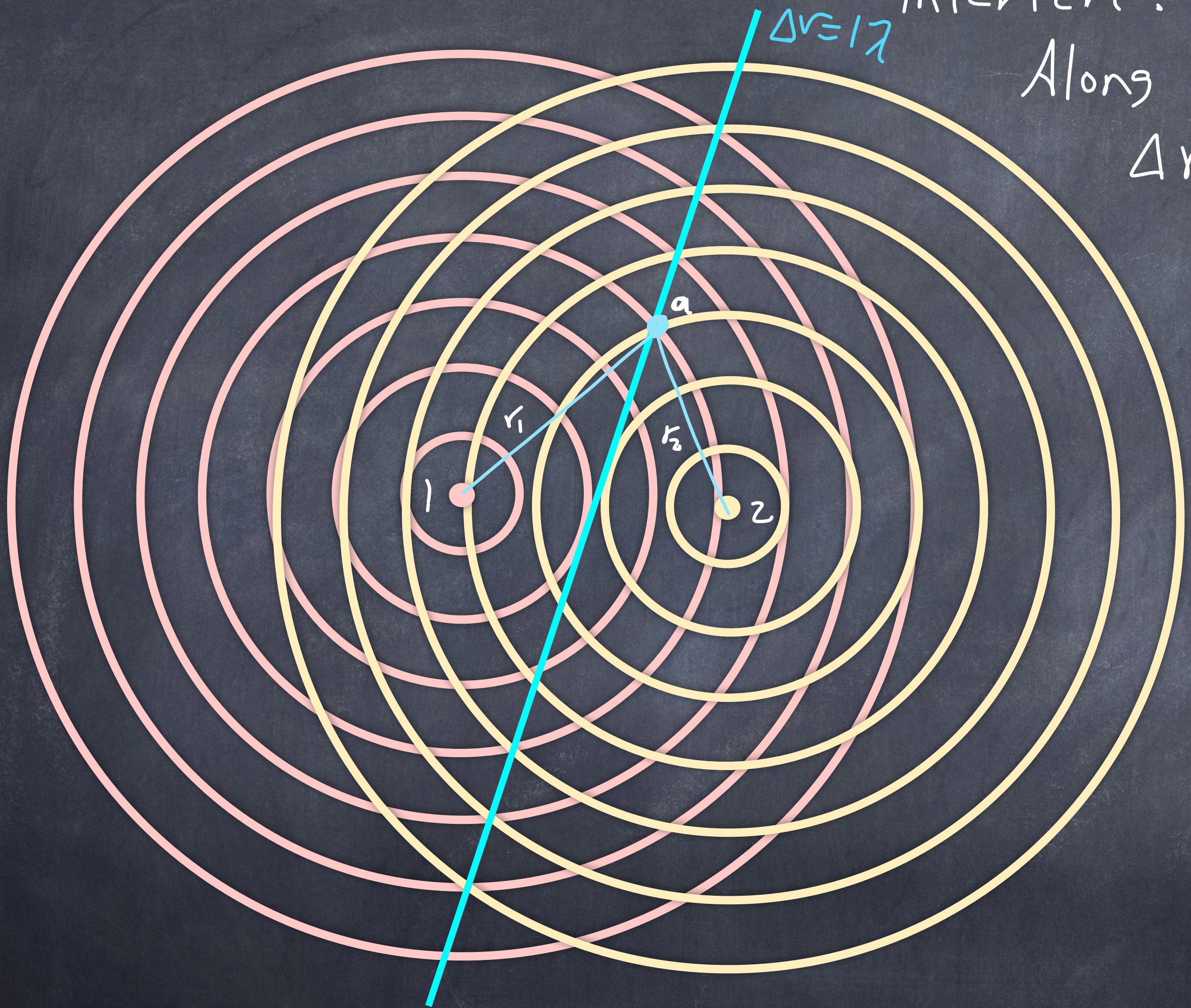
Along this line, where $r_1 = r_2$, the waves constructively interfere.

$$\Delta r = r_2 - r_1 = 0\lambda = 0$$



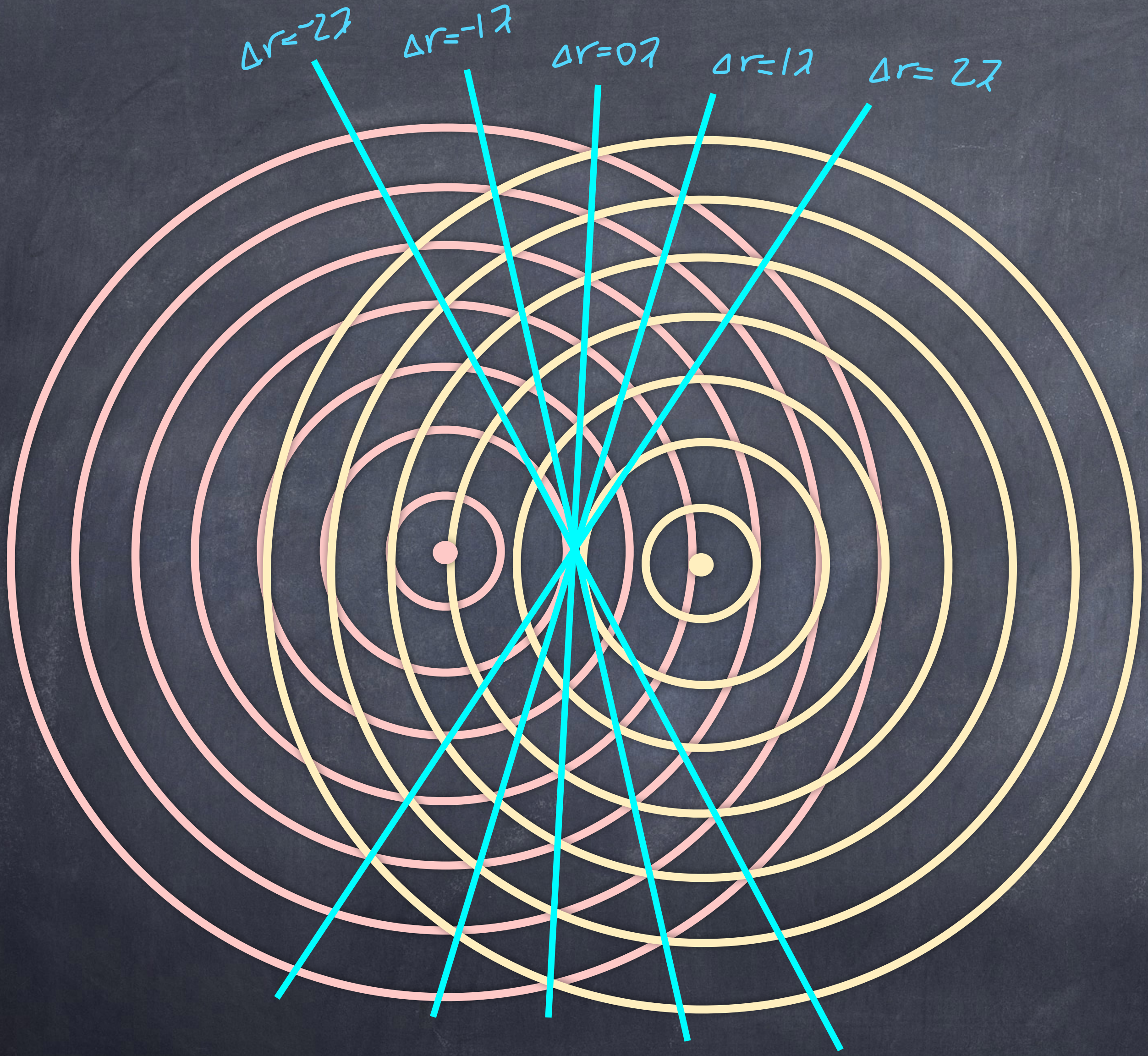
At point 3, the 3rd peak of one wave adds with the 3rd peak of other wave.

Along this line, the waves constructively interfere.



Along this line, $\Delta r = 1\lambda$

At point a, the 4th peak of source 1 adds with the 3rd peak of source 2.

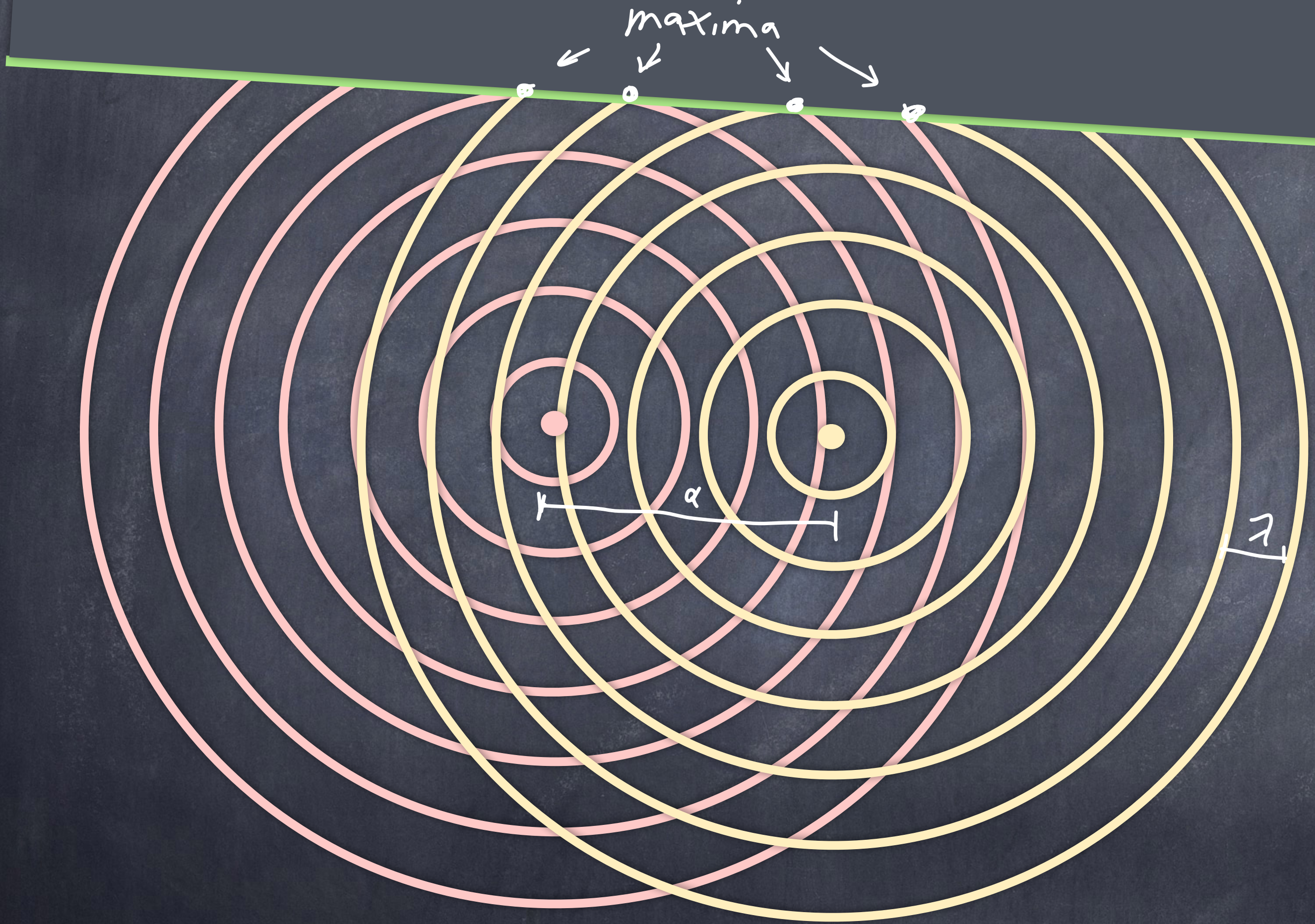


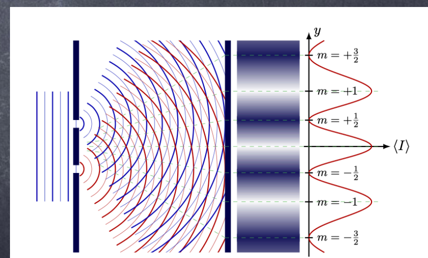
Along these lines, the waves add constructively

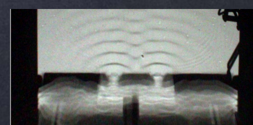
$$\Delta r = n\lambda$$

$$n = 0, \pm 1, \pm 2, \ldots$$

If we measure the sum of the waves along the green line, there will be constructive maxima at the intersection of the peaks from the 2 sources



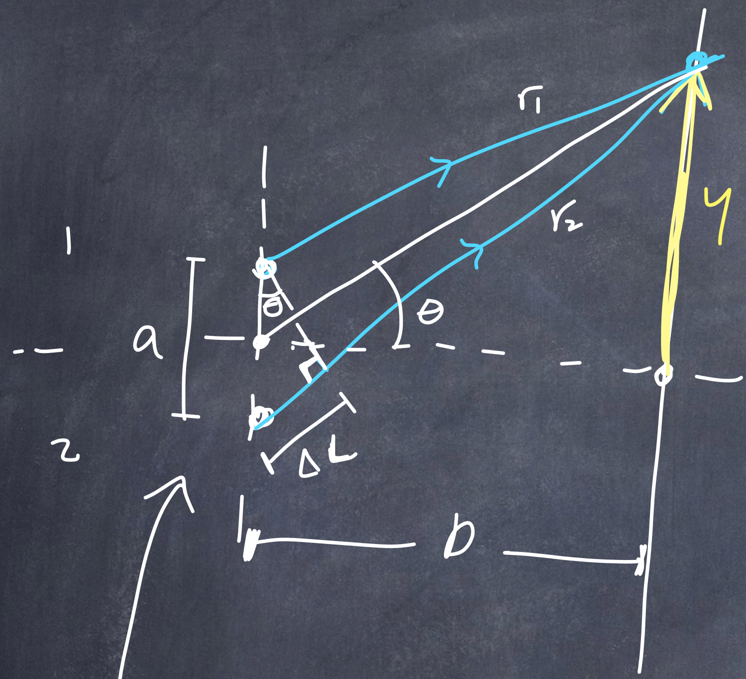




sources are a distance, a, apart

Calculate the location of the maxima. We measure at a distance, D.

2 coherent sources, a, apart.



ΔL = path difference from sources to point y,

$$\Delta L = r_2 - r_1$$

$$\tan\theta = \frac{y}{D}$$

If $D \gg a$, then we can use the small angle approximation:

$$\sin\theta \sim \underset{\uparrow \frac{y}{D}}{\tan\theta} \sim \theta$$ (radians)

From our small triangle
$$\sin\theta = \frac{\Delta L}{a}$$

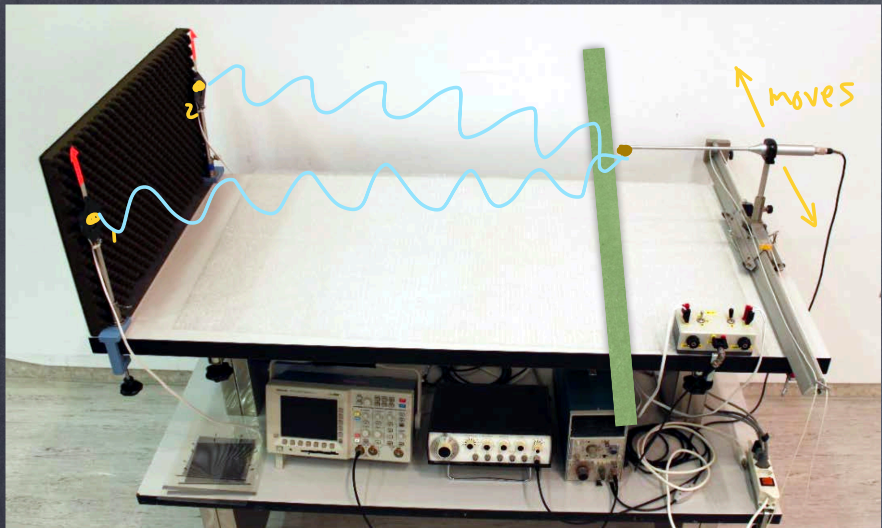
$$\Delta L = \frac{ya}{D}$$

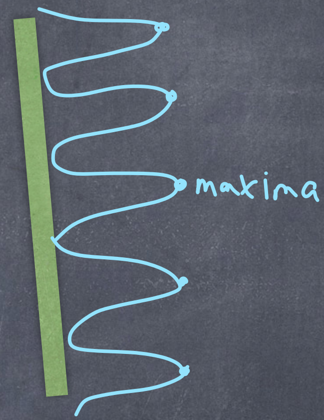
There are interference maxima when $\Delta L = n\lambda$ $\quad n = 0, \pm 1, \pm 2, \ldots$

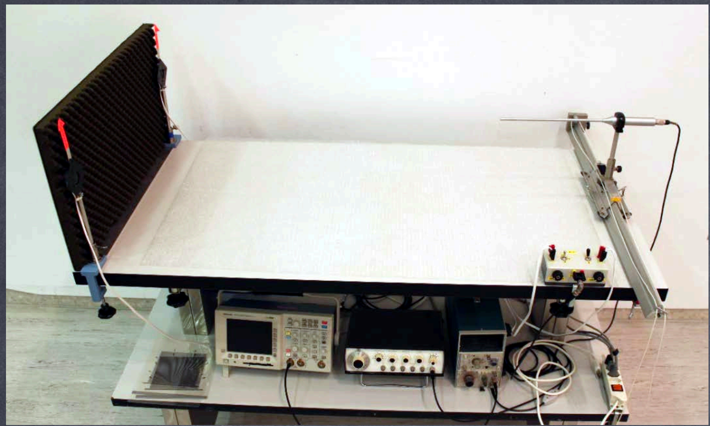
So we can set $\Delta L = n\lambda = \frac{ya}{D} \implies \boxed{y_n = \frac{n\lambda D}{a} \text{ location of maxima}}$

2 sources

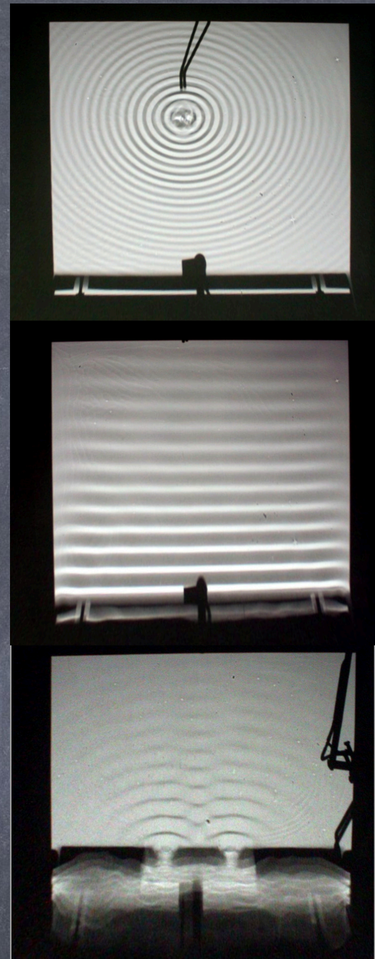
1 receiver







W110



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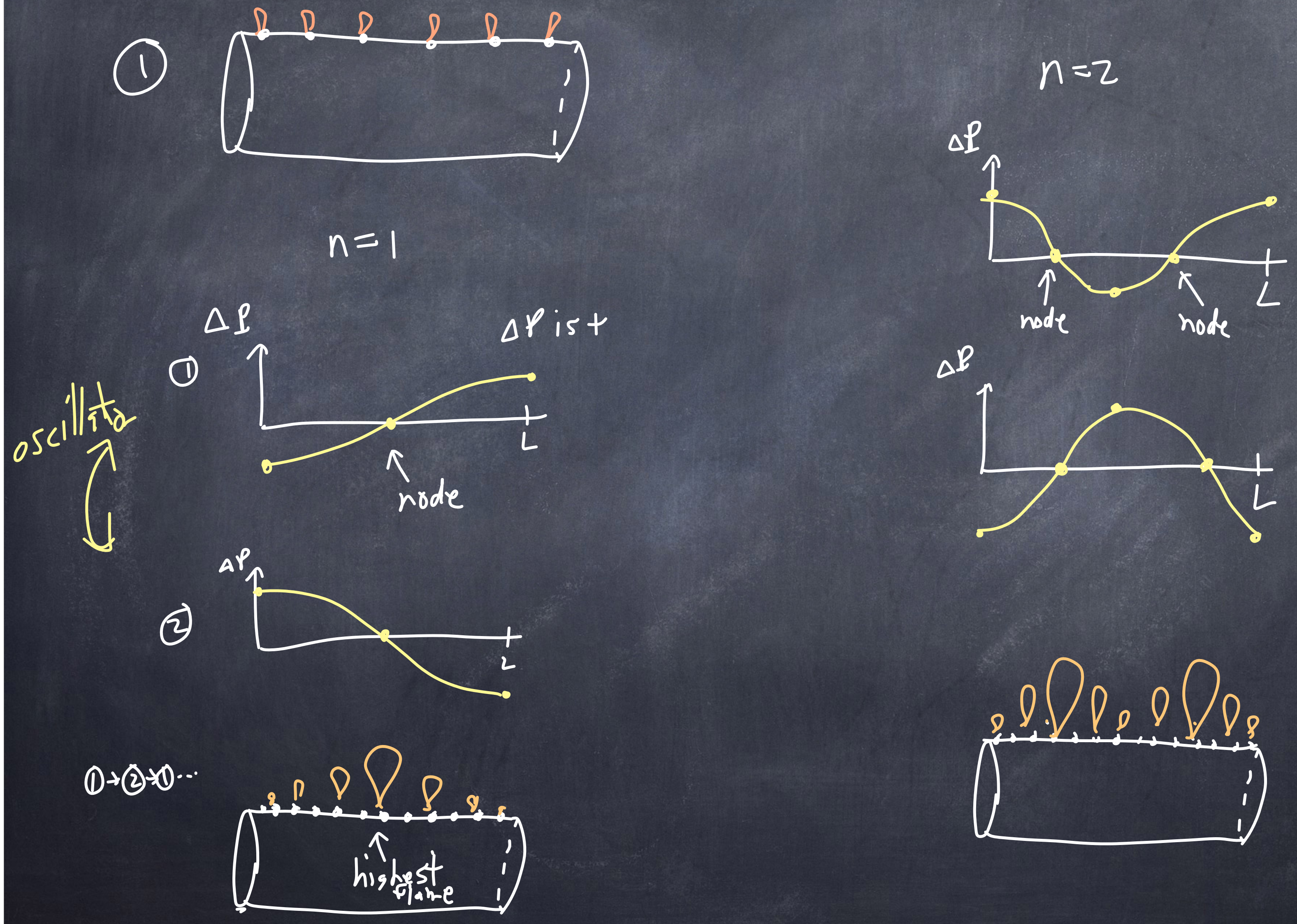
Ruben's flame tube



For quiet sounds, ΔP of the gas < P of the gas

From Bernoulli's equation, gas flow is proportional to square root of the pressure difference between inside + outside tube.

$$\text{flow} \sim \sqrt{P_{inside} - P_{outside}}$$

(The flow of gas out of the pipe)

$\Delta P_{maximal}$, anti-nodes produce lower flames (flow rate is lower)

$\Delta P = 0$, nodes, flow rate is higher

Part of the cycle, pressure is higher than average but part is lower. On average

This is why pressure is higher at nodes:

$$\sqrt{\text{Pressure difference at anti-nodes}} < \sqrt{\text{Pressure at nodes}}.$$