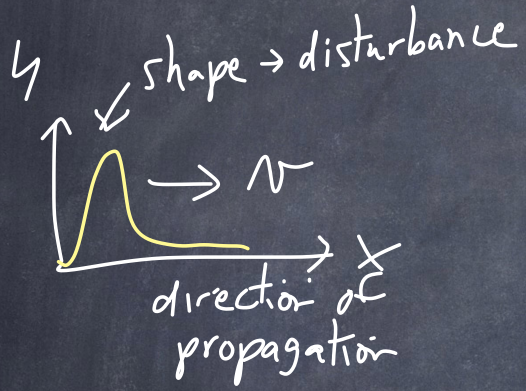
PHY117 HS2023

Week 11, Lecture 2
Nov. 29th, 2023
Prof. Ben Kilminster

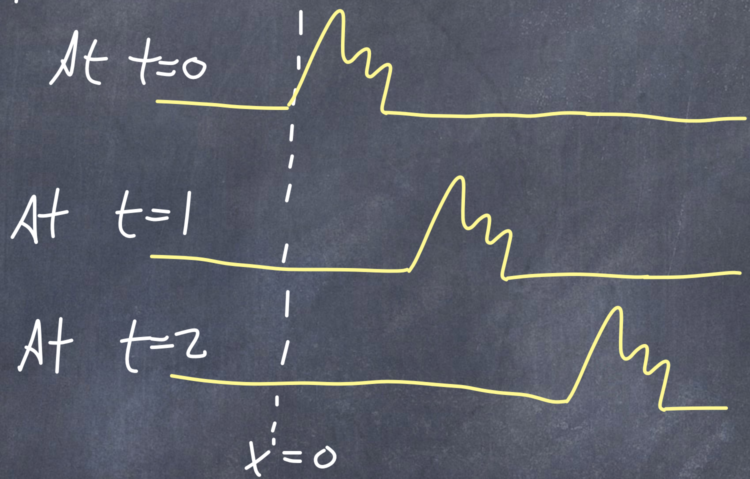
WAVES

WAVES:

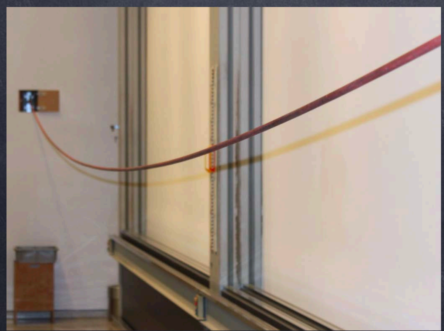
wave propagation → one makes a disturbance that propagates along some medium. ← (air, water, string, ...)



Transverse wave: disturbance $\perp$ propagation



The shape of the wave $y = f(x)$ at $t=0$ is the same at $t=1,2$



Transverse wave propagating at velocity v. It's value of y depends on x and t : $y = f(x, t)$

S reference frame (coordinate system x, y)

(a) Shape of wave at time $t = 0$ can be some function f in space variable x.

(b) The shape moved by a distance $v\Delta t$ at time $t = \Delta t$.

Figure 13.1: A transverse wave is a traveling wave that distorts a medium in the direction perpendicular to the direction of propagation.

At time $t=0$, the wave has a shape $y(x, t=0) = f(x)$, The wave travels with velocity v, but shape stays same. The peak at $x=a$ will move to $x = a + vt$ after time t.

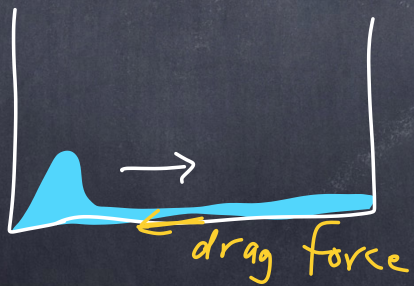
position

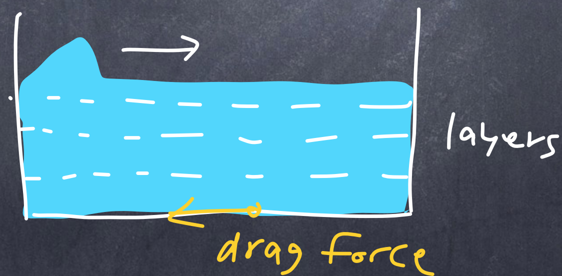
If we make a new reference frame S' with coordinates x', y', that moves at v, then y' is always $f(x)$, for any time t.

In general, we can transform between S' and S using a Galilean transformation: $x = x' + vt$, $y = y'$

Therefore, in the S frame, the wave can be described as $y(x,t) = f(x - vt)$ gives us the same shape $f(x)$ at $t=0$

Disturbance moving in water





waves move slower in shallow water due to the drag force

A wave moving to the right has a form like $y = y(x,t) = f(x - vt)$

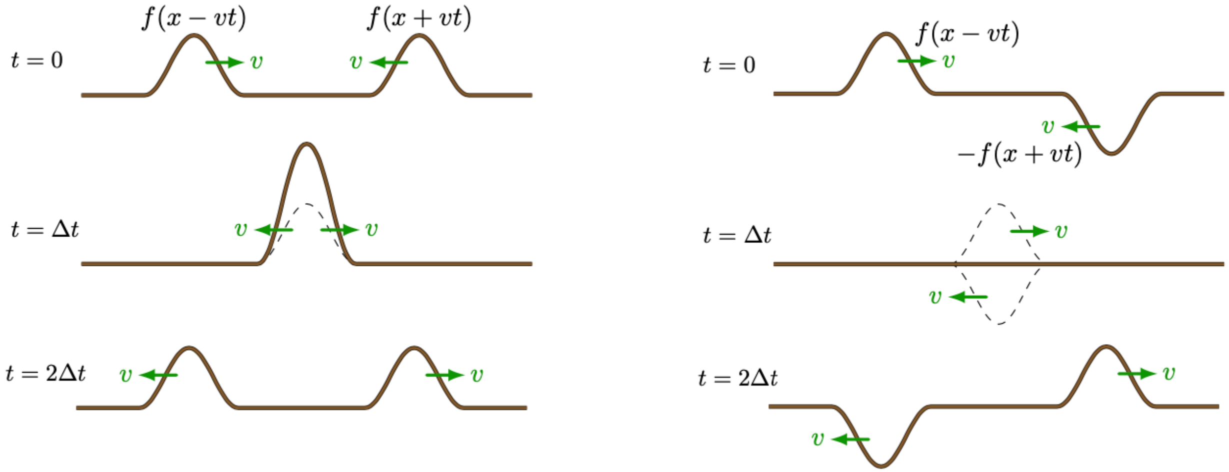
↑ any shape

Moving to the left, $y(x,t) = f(x + vt)$

If we had 2 waves, one to the left & one to the right, we can add them linearly

$$y(x,t) = y(x - vt) + y(x + vt)$$

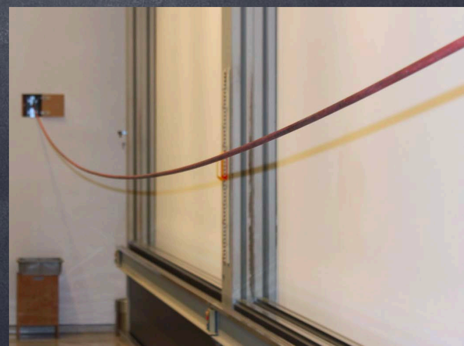
Superposition: is the addition of waves



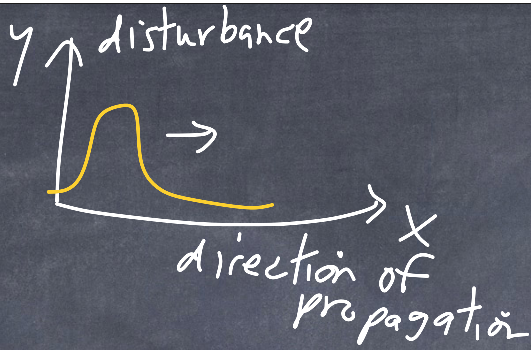
(a) Constructive interference happens when two oppositely waves meet on a string.

(b) Destructive interference. If the waves are the same but for a sign, they cancel completely.

Figure 13.5: Superposition between two oppositely travelling waves in the same medium is a simple linear sum.



Transverse wave:



disturbance $\perp$ direction it moves

Longitudinal wave

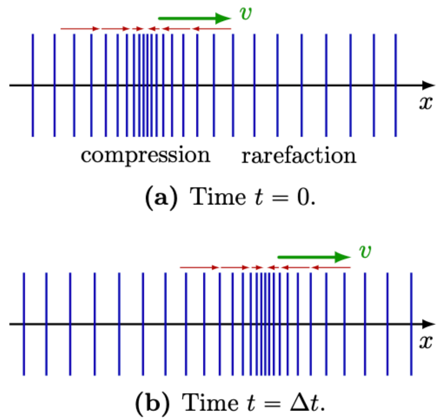
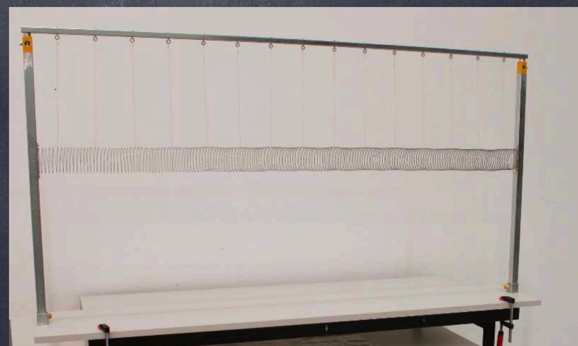


Figure 13.10: A traveling longitudinal wave is when the distortion happens along the direction of propagation, here shown as a local displacement.

disturbance $\parallel$ direction it moves

(sound waves are longitudinal waves)



Speed of a wave on a string] T: tension of string

μ: mass per length

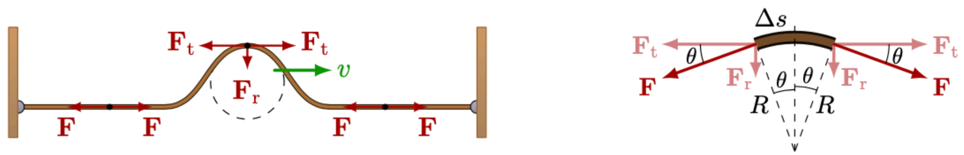
velocity of the disturbance

$$v = \sqrt{\frac{T}{\mu}}$$

← faster if tension is larger

← slower if mass per length is larger

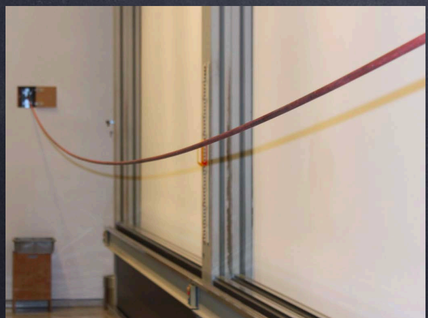
Derivation of this formula in script §13.1.2



(a) Forces on a string. All across the string, there is a constant tension $\mathbf{F}$.

(b) Small segment of length Δs experiences a tension $\mathbf{F}$ on either side.

Figure 13.3: The tension in a string is increased due to a disturbance.

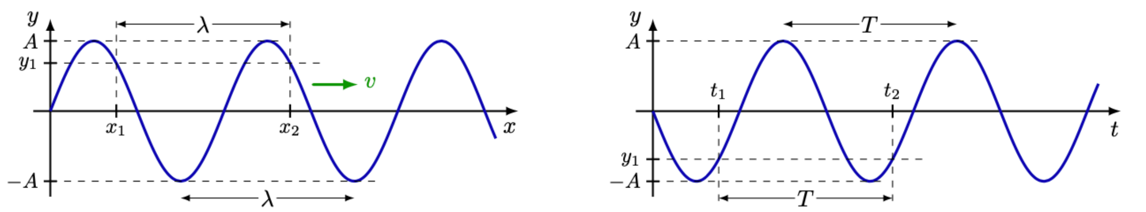


General form for a wave is $y(x,t) = f(x - vt)$

One type of wave is a sine wave:

$$\boxed{y(x,t) = A\sin(kx - \omega t)} \quad (1)$$

$t=0$ ↘ $\qquad\qquad$ $x=0$ ↘



(a) Whole wave in space at time $t = 0$, given by $y(x,0) = A\sin(kx)$.

(b) Local disturbance at position $x = 0$, given by $y(0,t) = -A\sin(\omega t)$.

Figure 13.2: A space and time slice of a travelling sine wave $y(x,t) = A\sin(kx - \omega t)$.

what is k? we see that the wave repeats every wavelength λ.

so for $x_2 = x_1 + \lambda$, $y(x_1, 0) = y(x_2, 0)$

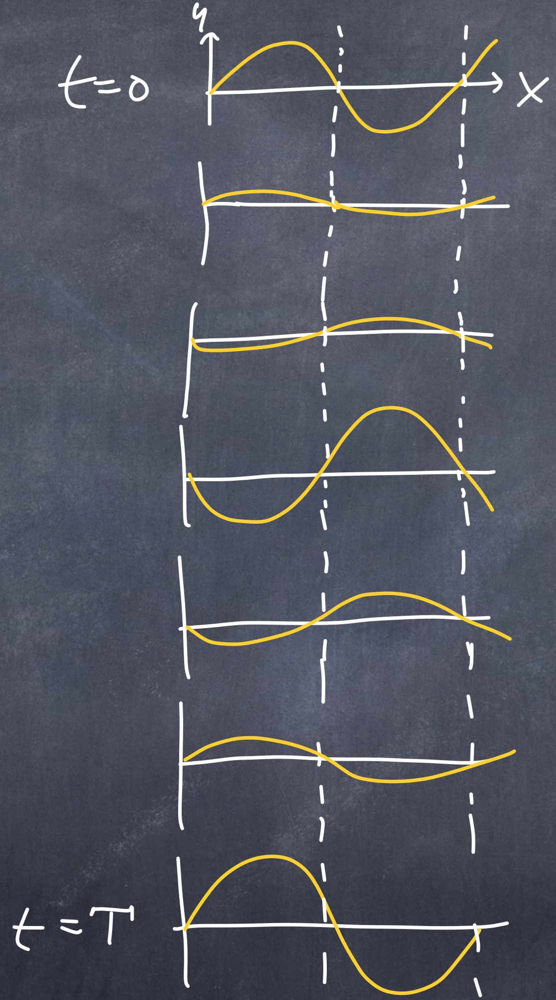
From (1) (with $t=0$), $A\sin(kx_1) = A\sin(kx_2) = A\sin(kx_1 + k\lambda)$

This happens only if $k\lambda = n(2\pi)$ where $n = 0, \pm 1, \pm 2, \ldots$

The smallest interval between repeating points is for $n=1$

so $k\lambda = 2\pi \Rightarrow \boxed{k = \frac{2\pi}{\lambda} \text{ wave number}}$

The shape of a wave changes in the same place, with time.



the wave repeats every time = T

the period

so when $x=0$

$\downarrow$

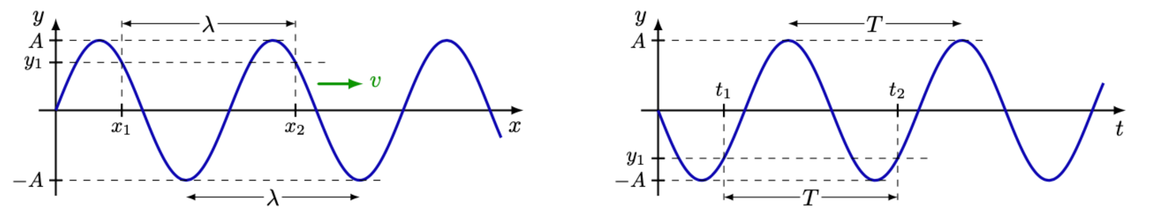


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Figure 13.2: A space and time slice of a travelling sine wave $y(x, t) = A\sin(kx - \omega t)$.

$$y(x,t) = A\sin(kx - \omega t)$$

when $x=0$, $y(0,t) = A\sin(-\omega t)$

Two moments in time, t_1 and $t_2 = t_1 + T$ must be separated by 2π

so $\omega(t_1 + T) = \omega t_1 + 2\pi$

and $\boxed{\omega = \frac{2\pi}{T} \quad \text{angular velocity}}$

Summary: For our wave, λ wavelength $[m]$

T period $[s]$

v velocity $\left[\frac{m}{s}\right]$

The velocity of our wave is

$$v = \frac{\lambda}{T} = f\lambda$$

$$f = \frac{1}{T} \quad \left[\frac{1}{s}\right] = [Hz]$$

wave number $= \quad k = \frac{2\pi}{\lambda} \quad \left[\frac{1}{m}\right]$

angular velocity $= \quad \omega = 2\pi f = \frac{2\pi}{T} \quad \left[\frac{1}{s}\right]$
(angular frequency)

$$v = \frac{\lambda}{T} = \frac{\frac{2\pi}{k}}{\frac{2\pi}{\omega}} = \frac{\omega}{k}$$

$$\boxed{v = \frac{\omega}{k}}$$

Example for a sine wave

$$y(x,t) = A\sin(kx - \omega t)$$

wave $\boxed{\text{function}}$ for a moving sine wave

$$y(x,t) = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

same formula with λ + T

wave equation $\quad \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad y(x,t)$

see derivation $\longrightarrow$ **13.2 Wave equation**

all wave functions satisfy the wave equation:

consider $f(x,t) = A\sin(kx-\omega t)$

$$\frac{\partial f}{\partial x} = Ak\cos(kx-\omega t), \quad \frac{\partial^2 f}{\partial x^2} = -Ak^2\sin(kx-\omega t)$$

$$\frac{\partial f}{\partial t} = -A\omega\cos(kx-\omega t), \quad \frac{\partial^2 f}{\partial t^2} = -A\omega^2\sin(kx-\omega t)$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

$$-\cancel{A}k^2\cancel{\sin(kx-\omega t)} = \frac{1}{v^2}\left(-\cancel{A}\omega^2\cancel{\sin(kx-\omega t)}\right)$$

$$\frac{k^2}{\omega^2} = \frac{1}{v^2} \Rightarrow v = \frac{\omega}{k}$$, which is true.

Standing waves – when we confine waves, the waves will reflect and combine by the superposition principle.

There are resonant frequencies and wavelengths in standing waves.

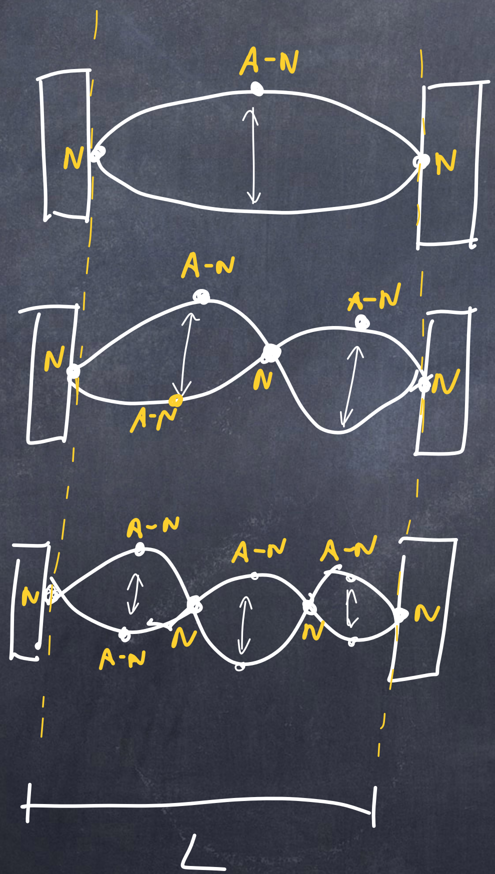
N : Nodes (points that don't move)

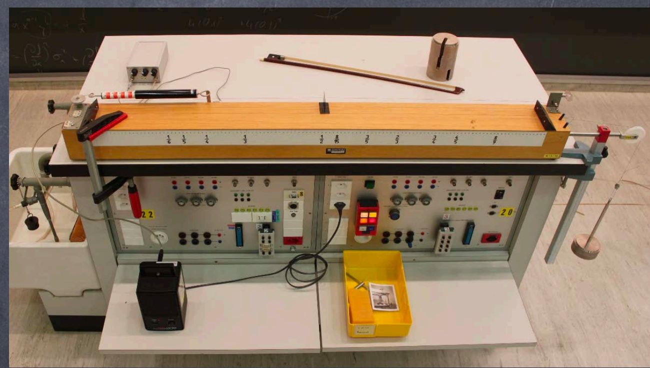
A-N : anti-nodes (points that move maximally)

Fundamental harmonic (first harmonic) $n = 1$

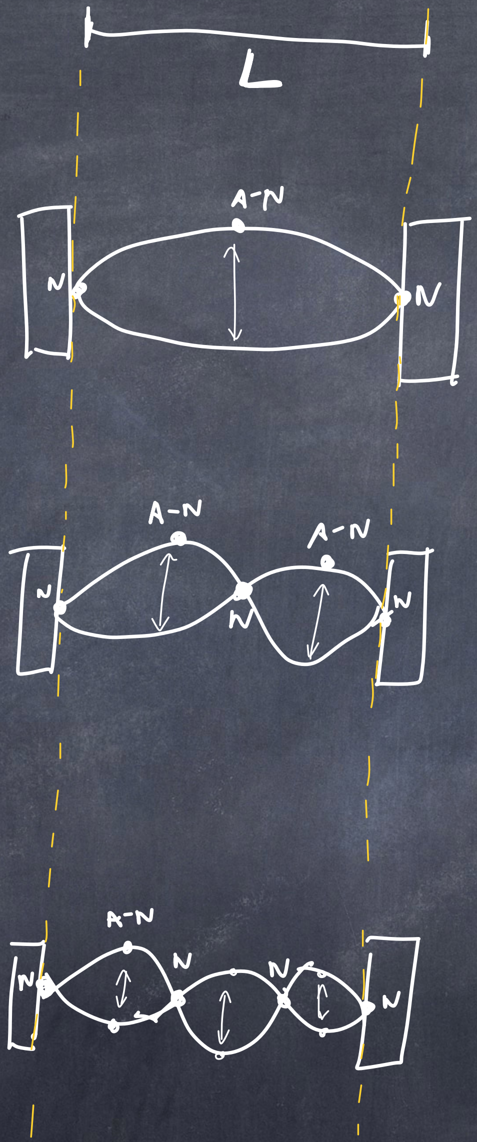
2nd harmonic $n = 2$

3rd harmonic $n = 3$





L: size of the space that the wave is confined one λ = Key



$n=1$

$\leftarrow \frac{1}{2}$ of a wave

$$L = \frac{\lambda}{2} \Rightarrow \lambda = 2L$$

$n=2$

$$L = \lambda$$

$n=3$

$1 + \frac{1}{2}$ waves

$$L = \frac{3}{2}\lambda$$

$$\lambda = \frac{2}{3}L$$

In general, the wavelengths of the harmonics are $\lambda_n = \frac{L}{n} 2$

$n = 1, 2, 3, \ldots$

The frequencies of the n^{th} harmonic $\quad v = f\lambda$

$$f_n = \frac{v}{\lambda_n} = \frac{v}{\frac{2L}{n}} = \frac{nv}{2L} \quad \text{for } n = 1, 2, 3, \ldots$$

For $n = 1$,
first harmonic $\quad f_1 = \frac{1 \cdot v}{2L} = \frac{v}{2L}$

for a string, $v = \sqrt{\frac{F}{\mu}}$ $\qquad f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$

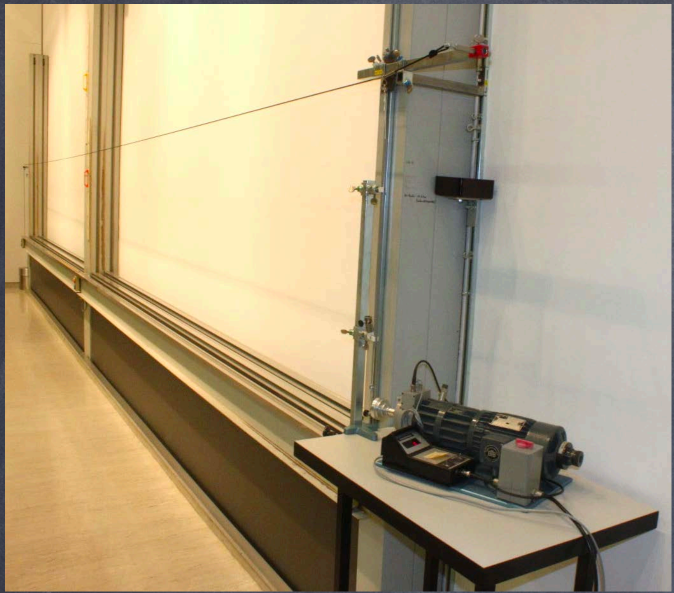
tension on string

mass per length of string

IF we know f_1, $f_n = n \cdot f_1$

$f_1 = \frac{v}{2L}$ $\qquad f_n = \frac{nv}{2L}$

All standing waves have a frequency that is a multiple of the first harmonic, f_1, depends on tension, length, $\frac{\text{mass}}{\text{length}}$



$f_n = n f_1$

First, we find f_1, the frequency of the first harmonic.

Then, $f_2 = 2 f_1$

$f_3 = 3 f_1$

...

standing wave functions

– consider the superposition of a wave moving to the left and to the right.

$$y_R = A\sin(kx - \omega t)$$

$$y_L = A\sin(kx + \omega t)$$

$k + \omega$ are both the same

$$y(x,t) = y_R + y_L = A\left[\sin(kx - \omega t) + \sin(kx + \omega t)\right]$$

substitution

$$\sin\theta_1 + \sin\theta_2 = 2\cos\tfrac{1}{2}(\theta_1 - \theta_2)\sin\tfrac{1}{2}(\theta_1 + \theta_2)$$

we set $\theta_1 = kx + \omega t$

and $\theta_2 = kx - \omega t$

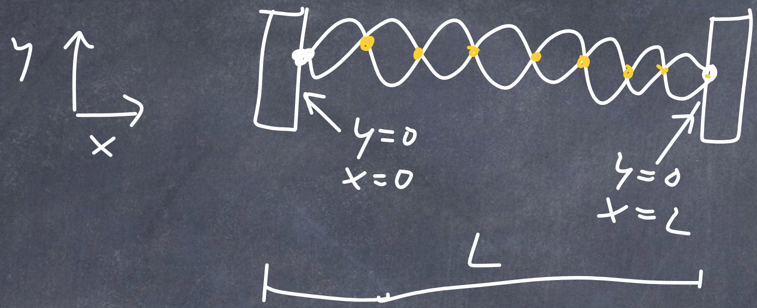
add these: $\theta_1 + \theta_2 = 2kx$

$\theta_1 - \theta_2 = 2\omega t$

$$\boxed{y(x,t) = 2A(\cos\omega t)(\sin kx)}$$

formula for a standing wave with 2 fixed ends (nodes)

Now only some ω + k work for standing waves.
we consider the boundary conditions



$y=0$ when $x=0$, or $x=L$

when does $y=0$? Happens when $\sin kL = 0$

so $k_n L = n\pi$ $\quad n=1,2,3,\ldots$

since $\lambda = \frac{2\pi}{k}$

$$\boxed{k_n = \frac{n\pi}{L} \quad \text{for } n=1,2,\ldots}$$

allowed wave numbers

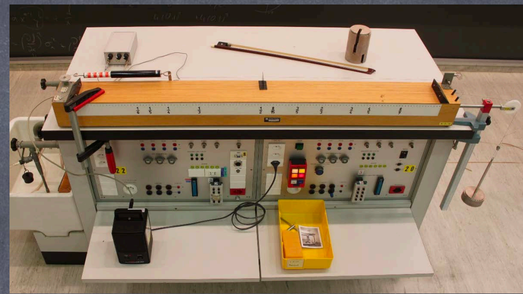
we know $f_n = \frac{v}{\lambda_n} = \frac{vn}{2L}$ $\quad n=1,2,3,\ldots$

$$\boxed{f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}}$$

$f_n = n \cdot f_1$

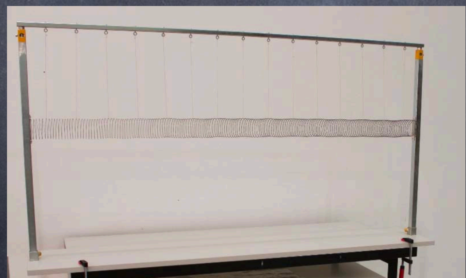
since $\omega_n = 2\pi f_n$

$$\boxed{\omega_n = 2\pi n f_1 \quad \text{for } n=1,2,3,}$$



W14

W18



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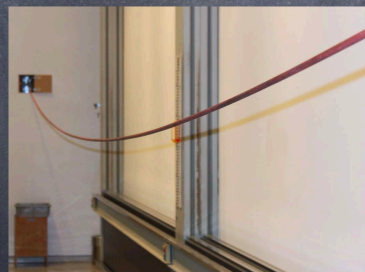

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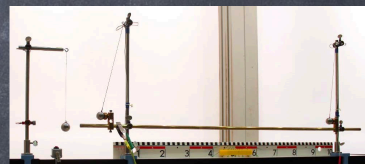

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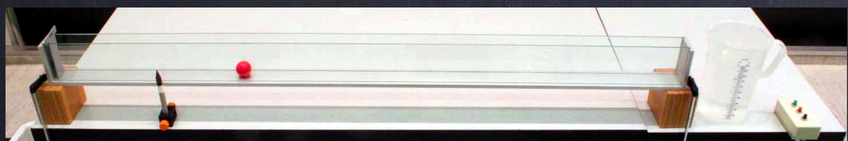


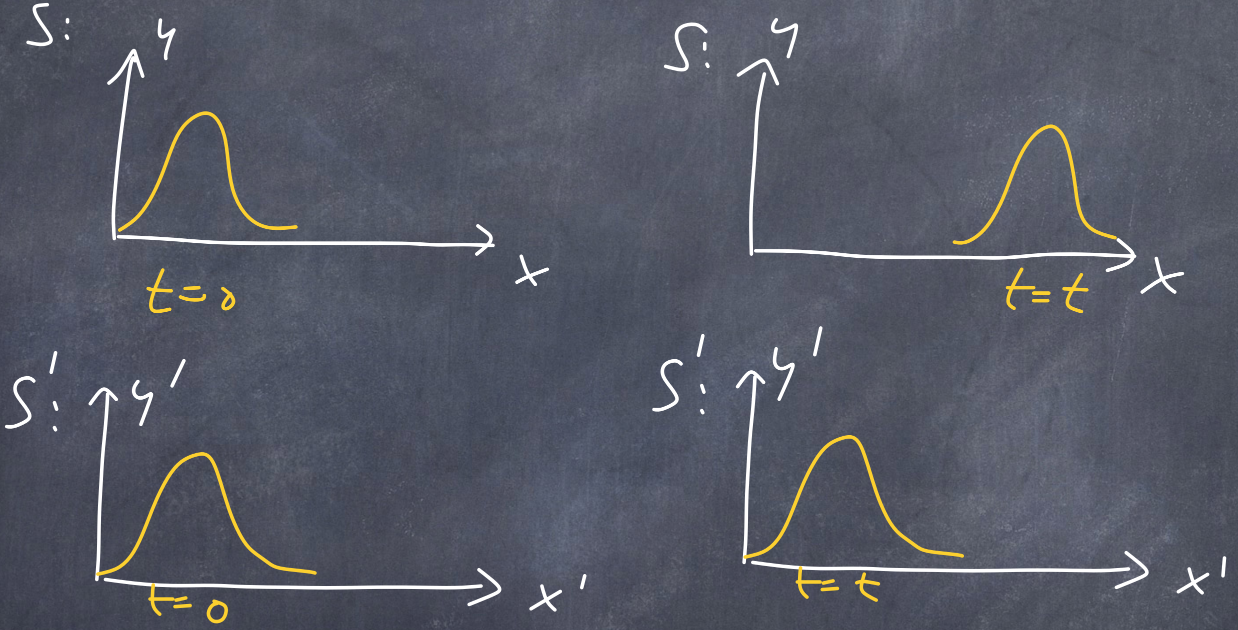
W31

W33

we define 2 coordinate systems (or rest frames):

S: not moving

S': moving with same velocity as wave



S' has velocity v

$y' = y$

$x' = x - vt$

So we see that we can describe our wave in the S frame by transforming its shape back to time $t=0$: $y'(x',t) = y(x - vt)$

wave function for wave moving to the right