Non-linearity $x_n(1-x_n)$

$\hookrightarrow$ fractal behaviour and Chaos

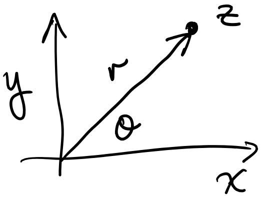
Complex Iterator (using complex numbers)

$$i^2 = -1$$

$$z = (x + iy)$$

Real   Imaginary

$$z = r e^{i\theta}$$



let $w = u + iv$

Addition $z + w = (x+u) + i(y+v)$

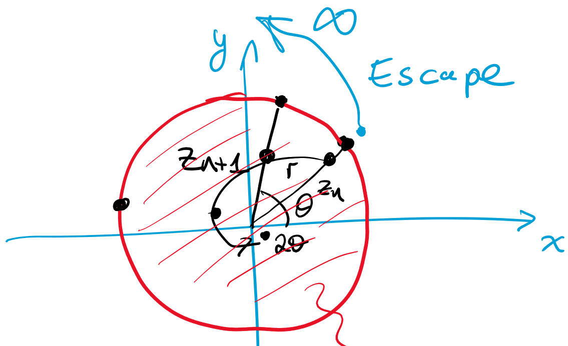
Multiply $z \cdot w$

$$x \cdot u + ixv + iyu + \underbrace{i^2 yv}_{-yv}$$

$$(x \cdot u - yv) + i(xv + yu)$$

$$z^2 = (re^{i\theta})^2 = r^2 e^{i(2\theta)}$$

Simplest case would be $\boxed{z_{n+1} = z_n^2}$

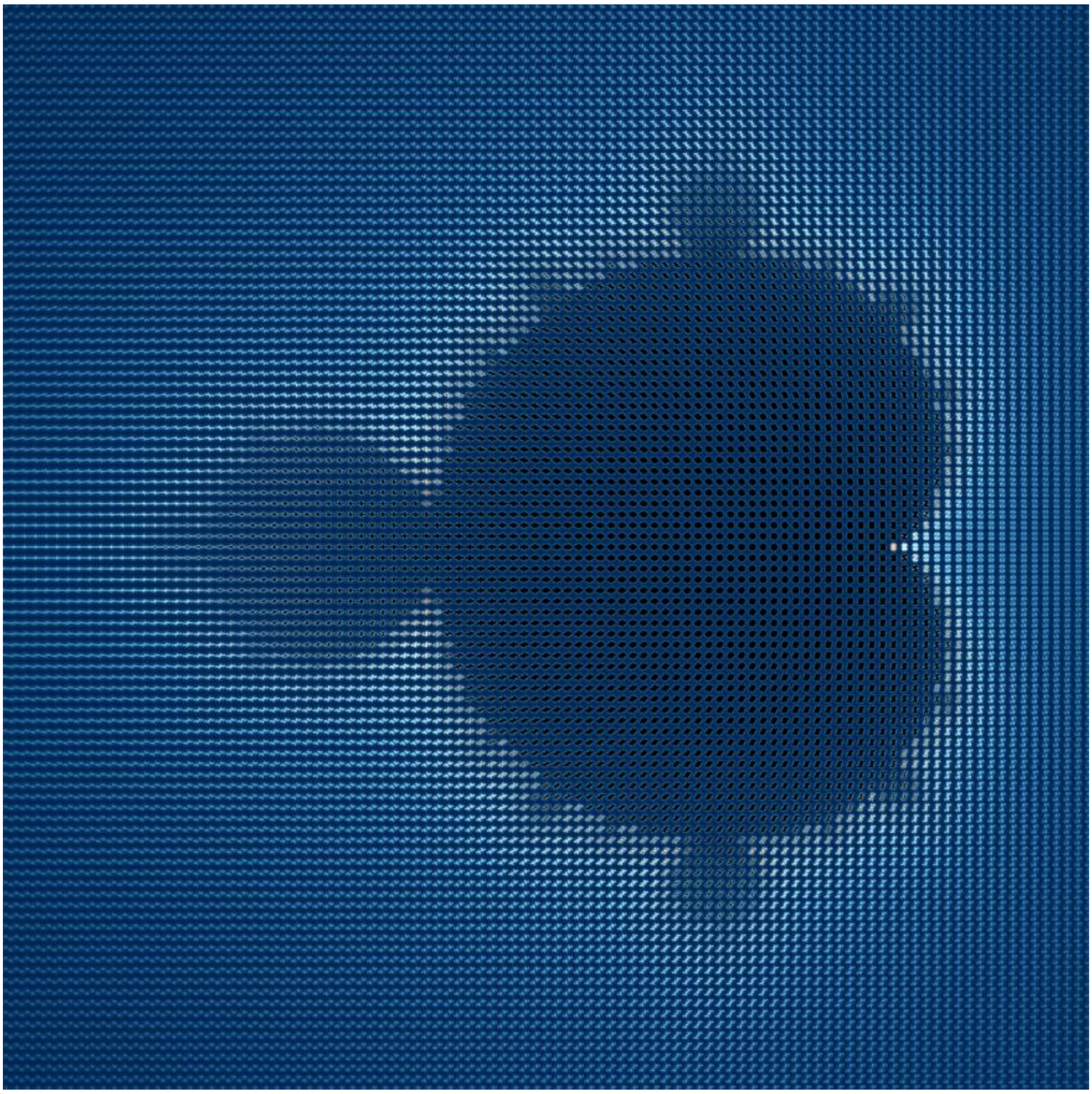


Prisoners $\longrightarrow$ Julia Set

$$z_{n+1} = z_n^2 + C$$

Julia Set for
$c = -0.5 + 0.5i$
Connected

$c = ?$
Unconnected



A grid of Julia Sets for different values of c

the Mandelbrot Set $M = \{c \in \mathbb{C} \mid J_c \text{ is connected}\}$

of c

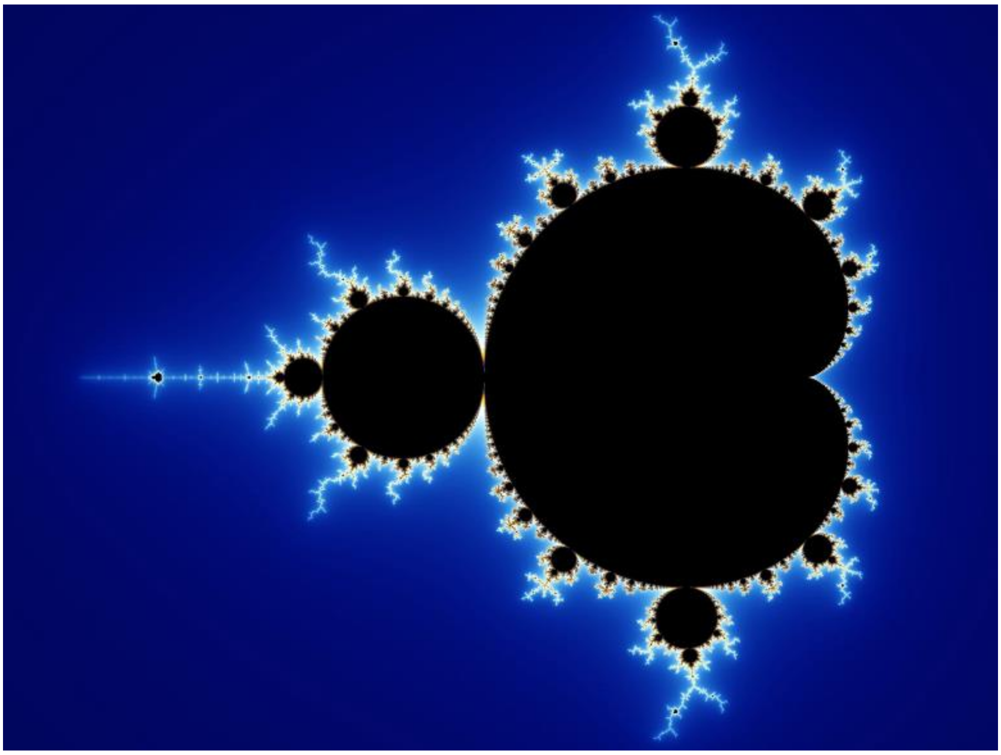
Mandelbrot Set $\quad M = \{ c \in \mathbb{C} \mid J_c \text{ is connected} \}$

Another definition:

$$M = \{ z_0 = c \in \mathbb{C} \mid z_{n+1} = z_n^2 + c < \infty \}$$

if $|z_n| > r(c)$ then it escapes to ∞ and not a part of M.

$$r(c) = \max(|c|, 2)$$



M

Colors: according to how many steps to escape. How "close" you are to the Mandelbrot set.

---

for all pixels on screen (values of c):

```
k = 0
z = c
while (k < 100)
    if (|z| > r(c)) then
        draw point with color(k)
        (c ∉ M)
```

draw point with color(k)
mark ($c \notin M$)

$z = z * z + c$

$k = k + 1$

draw point with color("black")
mark ($c \in M$)

Ordinary Differential Equations (ODEs)

$$\frac{d^2 y}{d\boxed{x}^2} + q(x)\frac{dy}{d\boxed{x}} = r(x) \quad 2^{nd}\text{-order}$$

rewrite

$$\boxed{\begin{aligned} \frac{dy}{dx} &= z(x) \\ \frac{dz}{dx} + q(x)\cdot z(x) &= r(x) \end{aligned}}$$

2 times a 1^{ST} order ODE

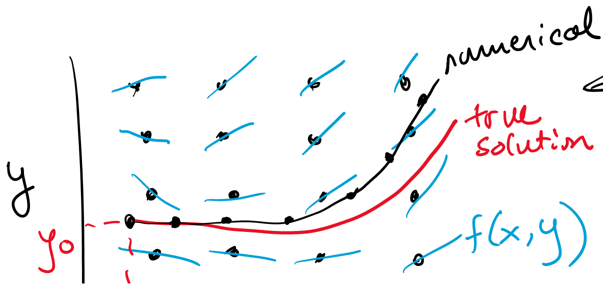
In general we can write

$$\frac{dy_i}{dx} = f_i(x, y_0, y_1, \dots y_{N-1})$$

$$i = 0 \,..\, N-1$$

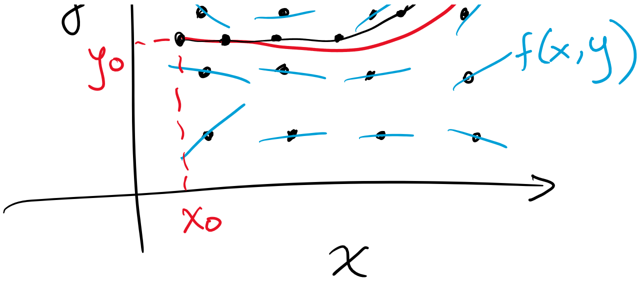
$$\boxed{\frac{d\underline{y}}{dx} = \underline{f}(x, \underline{y})}$$

What does it mean?



$$\frac{dy}{dx} = f(x,y)$$

Goal is to solve



Goal is to solve $y(x)$

$y_0 = y(x_0)$ Boundary condition

Different boundary conditions

$y_0 = y(t_0)$ Initial condition

Dirchlet B.C.

$$y_i(x_{start}) = y_i^{start}$$

Given

von Neumann B.C.

$$\frac{d\, y_i(x_{start})}{dx} = \text{Slope given somewhere}$$